# 09 Exponentiële en logaritmische functies

## 9.1 Rekenregels en grafieken

### bladzijde 8

**1** **a** $2^{x+5} = 2^x \cdot 2^5 = 2^5 \cdot 2^x = 32 \cdot 2^x$

**b** $2^{x-4} = 2^x \cdot 2^{-4} = 2^{-4} \cdot 2^x = \frac{1}{16} \cdot 2^x$

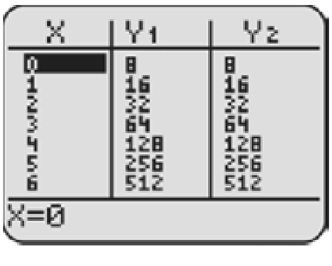
**c** $2^{x+\frac{1}{2}} = 2^x \cdot 2^{\frac{1}{2}} = 2^{\frac{1}{2}} \cdot 2^x = \sqrt{2} \cdot 2^x$

**2** **a** $y = 2^x$
$\downarrow$ translatie (4, 0)
$y = 2^{x-4}$
$\downarrow$ translatie (0, 3)
$y = 3 + 2^{x-4}$

**b** $y = 2^x$
$\downarrow$ translatie (–1, 0)
$y = 2^{x+1}$
$\downarrow$ verm. t.o.v. de $x$-as met 3
$y = 3 \cdot 2^{x+1}$

**c** $y = 2^x$
$\downarrow$ verm. t.o.v. de $y$-as met $\frac{1}{4}$
$y = 2^{4x}$
$\downarrow$ translatie (0, 5)
$y = 5 + 2^{4x}$

**3** **a**



| X | Y1 | Y2 |
|---|---|---|
| 0 | 8 | 8 |
| 1 | 16 | 16 |
| 2 | 32 | 32 |
| 3 | 64 | 64 |
| 4 | 128 | 128 |
| 5 | 256 | 256 |
| 6 | 512 | 512 |

X=0

De uitkomsten van $y_1$ en $y_2$ zijn gelijk.

**b** $2^{x+3} = 2^x \cdot 2^3 = 2^3 \cdot 2^x = 8 \cdot 2^x$

### bladzijde 9

**4** **a** $y = 2^x$
$\downarrow$ translatie (5, 0)
$y = 2^{x-5}$
Er geldt $2^{x-5} = 2^x \cdot 2^{-5} = 2^{-5} \cdot 2^x = \frac{1}{32} \cdot 2^x$

Dus de vermenigvuldiging t.o.v. de $x$-as met $\frac{1}{32}$ levert dezelfde beeldfiguur op.

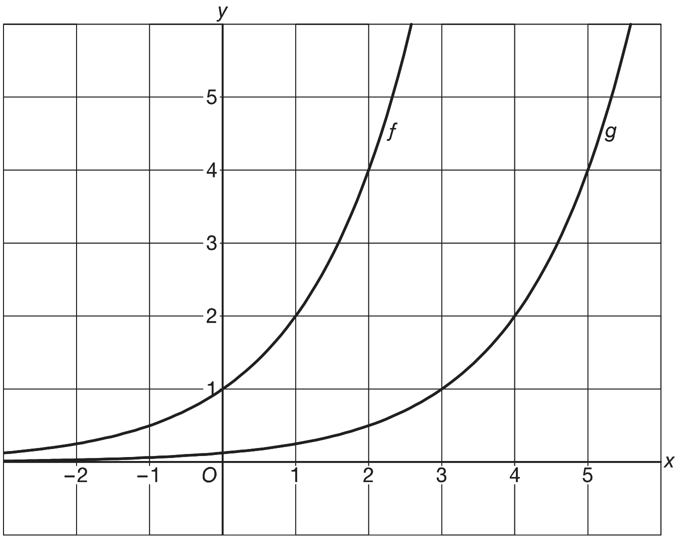
**b** $y = 4^x$
$\downarrow$ verm. t.o.v. de $x$-as met 2
$y = 2 \cdot 4^x$

Er geldt $2 \cdot 4^x = \sqrt{4} \cdot 4^x = 4^{\frac{1}{2}} \cdot 4^x = 4^{x+\frac{1}{2}}$

Dus de translatie $(-\frac{1}{2}, 0)$ levert dezelfde beeldfiguur op.

**5** **a** $g(x) = 2^{x-3}$

**b**




**c** $2^{x-3} = 2^x \cdot 2^{-3} = 2^{-3} \cdot 2^x = \frac{1}{8} \cdot 2^x$

Ja, de vermenigvuldiging ten opzichte van de $x$-as met $\frac{1}{8}$.
Ten opzichte van de $y$-as is zo'n vermenigvuldiging niet mogelijk. Je kunt bijvoorbeeld het punt (0, 1) niet zo vermenigvuldigen dat het beeld (3, 1) is.

**d** $y = 2^{x-3}$
$\downarrow$ verm. t.o.v. de $x$-as met $\frac{1}{4}$
$y = \frac{1}{4} \cdot 2^{x-3}$

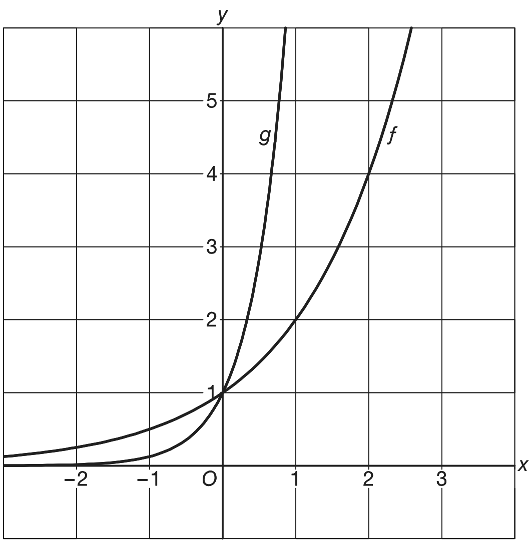
Er geldt $\frac{1}{4} \cdot 2^{x-3} = 2^{-2} \cdot 2^{x-3} = 2^{x-5}$

$$\left.\begin{array}{l} h(x) = 2^{x-5} \\ h(x) = 2^{x+p} + q \end{array}\right\} p = -5 \ \wedge \ q = 0$$

## bladzijde 10

**6** **a** $y = 2^x$
$\downarrow$ verm. t.o.v. de $y$-as met $\frac{1}{3}$
$y = 2^{3x}$

**b**




**c** Beide grafieken gaan door (0, 1).
Bij een vermenigvuldiging t.o.v. de $x$-as zou de factor dus één moeten zijn.
Dat is onmogelijk.
Een translatie is niet mogelijk omdat de grafieken niet dezelfde vorm hebben.

**d** $y = 2^{3x}$

$\downarrow$ translatie (1, 4)

$y = 2^{3(x-1)} + 4$

Er geldt $2^{3(x-1)} + 4 = (2^3)^{x-1} + 4 = 8^{x-1} + 4$

$\left.\begin{array}{l} h(x) = 8^{x-1} + 4 \\ h(x) = a^{x+p} + q \end{array}\right\} a = 8 \wedge p = -1 \wedge q = 4$

**7** **a** $y = (\frac{1}{2})^x$

$\downarrow$ verm. t.o.v. de $x$-as met 4

$y = 4 \cdot (\frac{1}{2})^x$

Dus de vermenigvuldiging t.o.v. de $x$-as met 4.

**b** Er geldt $4 \cdot (\frac{1}{2})^x = (\frac{1}{2})^{-2} \cdot (\frac{1}{2})^x = (\frac{1}{2})^{x-2}$

Dus de translatie (2, 0).

**c** Er geldt $4^x = ((\frac{1}{2})^{-2})^x = (\frac{1}{2})^{-2x}$

Dus de vermenigvuldiging t.o.v. de $y$-as met $-\frac{1}{2}$.

**d** $y = 4 \cdot (\frac{1}{2})^x$

$\downarrow$ verm. t.o.v. de $x$-as $\frac{1}{4}$

$y = (\frac{1}{2})^x$

$\downarrow$ verm. t.o.v. de $y$-as met $-\frac{1}{2}$

$y = (\frac{1}{2})^{-2x} = ((\frac{1}{2})^{-2})^x = 4^x$

De transformaties zijn verm. t.o.v. de $x$-as met $\frac{1}{4}$ en verm. t.o.v. de $y$-as met $-\frac{1}{2}$.

**e** $y = 4 \cdot (\frac{1}{2})^x$

$\downarrow$ translatie (3, 4)

$y = 4 \cdot (\frac{1}{2})^{x-3} + 4$

Er geldt $4 \cdot (\frac{1}{2})^{x-3} + 4 = 4 \cdot (\frac{1}{2})^x \cdot (\frac{1}{2})^{-3} + 4 = 4 \cdot (\frac{1}{2})^{-3} \cdot (\frac{1}{2})^x + 4 = 4 \cdot 8 \cdot (\frac{1}{2})^x + 4 = 32 \cdot (\frac{1}{2})^x + 4$

$\left.\begin{array}{l} j(x) = 32 \cdot (\frac{1}{2})^x + 4 \\ j(x) = a \cdot (\frac{1}{2})^x + b \end{array}\right\} a = 32 \wedge b = 4$

**8** **a** ${}^2\log(32) = {}^2\log(2^5) = 5$

**b** ${}^3\log(\frac{1}{9}) = {}^3\log(3^{-2}) = -2$

**c** ${}^{\frac{1}{2}}\log(8) = {}^{\frac{1}{2}}\log((\frac{1}{2})^{-3}) = -3$

**d** ${}^5\log(25\sqrt{5}) = {}^5\log(5^2 \cdot 5^{\frac{1}{2}}) = {}^5\log(5^{2\frac{1}{2}}) = 2\frac{1}{2}$

## bladzijde 11

**9** **a** $4 = {}^3\log(3^4) = {}^3\log(81)$

$3 = {}^3\log(3^3) = {}^3\log(27)$

$\frac{1}{3} = {}^3\log(3^{\frac{1}{3}}) = {}^3\log(\sqrt[3]{3})$

$-2 = {}^3\log(3^{-2}) = {}^3\log(\frac{1}{9})$

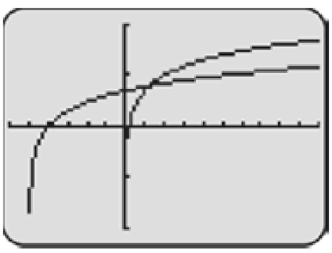
**b** $2 = {}^{\frac{1}{4}}\log((\frac{1}{4})^2) = {}^{\frac{1}{4}}\log(\frac{1}{16})$

$-1 = {}^{\frac{1}{4}}\log((\frac{1}{4})^{-1}) = {}^{\frac{1}{4}}\log(4)$

$-3 = {}^{\frac{1}{4}}\log((\frac{1}{4})^{-3}) = {}^{\frac{1}{4}}\log(64)$

$\frac{1}{2} = {}^{\frac{1}{4}}\log((\frac{1}{4})^{\frac{1}{2}}) = {}^{\frac{1}{4}}\log(\frac{1}{2})$

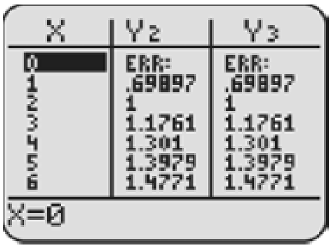
**10** **a**



$y = \log(x)$
$\downarrow$ translatie $(-5, 0)$
$y_1 = \log(x + 5)$

$y = \log(x)$
$\downarrow$ translatie $(0, \log(5))$
$y_2 = \log(x) + \log(5)$
$y = \log(x)$
$\downarrow$ verm. t.o.v. de $y$-as met $\frac{1}{5}$
$y_3 = \log(5x)$

**b**

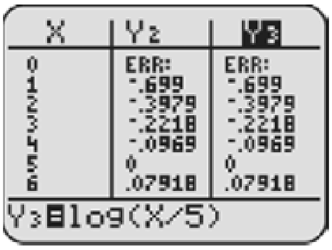


| X | Y2 | Y3 |
|---|---|---|
| 0 | ERR: | ERR: |
| 1 | .69897 | .69897 |
| 2 | 1 | 1 |
| 3 | 1.1761 | 1.1761 |
| 4 | 1.301 | 1.301 |
| 5 | 1.3979 | 1.3979 |
| 6 | 1.4771 | 1.4771 |



Uit de tabellen volgt dat $y_2 = y_3$.

**11** **a**

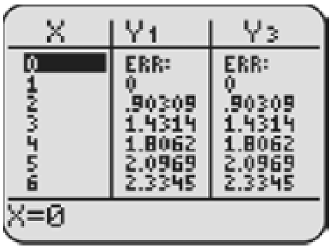


| X | Y2 | Y3 |
|---|---|---|
| 0 | ERR: | ERR: |
| 1 | -.699 | -.699 |
| 2 | -.3979 | -.3979 |
| 3 | -.2218 | -.2218 |
| 4 | -.0969 | -.0969 |
| 5 | 0 | 0 |
| 6 | .07918 | .07918 |



$y_2$ en $y_3$ komen op hetzelfde neer.

**b**

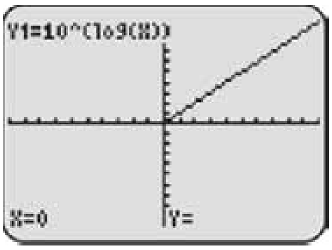


| X | Y1 | Y3 |
|---|---|---|
| 0 | ERR: | ERR: |
| 1 | 0 | 0 |
| 2 | .90309 | .90309 |
| 3 | 1.4314 | 1.4314 |
| 4 | 1.8062 | 1.8062 |
| 5 | 2.0969 | 2.0969 |
| 6 | 2.3345 | 2.3345 |



$y_1$ en $y_3$ komen op hetzelfde neer.

**c**




$10^{\log(x)} = x$ voor $x > 0$.

**12** **a** $2^{^2\log(32)} = 2^5 = 32$

**b** $3^{^3\log(81)} = 3^4 = 81$

**c** $4^{^4\log(\frac{1}{16})} = 4^{-2} = \frac{1}{16}$

**d** $5^{^5\log(25\sqrt{5})} = 5^{2\frac{1}{2}} = 25\sqrt{5}$

## bladzijde 13

**13** **a** ${}^2\log(6) + {}^2\log(10) = {}^2\log(6 \cdot 10) = {}^2\log(60)$

**b** ${}^3\log(30) - {}^3\log(6) = {}^3\log(\frac{30}{6}) = {}^3\log(5)$

**c** $2 \cdot {}^5\log(3) + {}^5\log(0{,}5) = {}^5\log(3^2) + {}^5\log(\frac{1}{2}) = {}^5\log(9) + {}^5\log(\frac{1}{2}) = {}^5\log(9 \cdot \frac{1}{2}) = {}^5\log(4\frac{1}{2})$

**d** $^{\frac{1}{2}}\log(15) - 4 \cdot {}^{\frac{1}{2}}\log(3) = {}^{\frac{1}{2}}\log(15) - {}^{\frac{1}{2}}\log(3^4) = {}^{\frac{1}{2}}\log(15) - {}^{\frac{1}{2}}\log(81) = {}^{\frac{1}{2}}\log(\frac{15}{81}) = {}^{\frac{1}{2}}\log(\frac{5}{27})$

**e** $-2 \cdot {}^4\log(6) + {}^4\log(12) = {}^4\log(6^{-2}) + {}^4\log(12) = {}^4\log(\frac{1}{36}) + {}^4\log(12) = {}^4\log(\frac{12}{36}) = {}^4\log(\frac{1}{3})$

**f** $\log(50) - 2 \cdot \log(5) = \log(50) - \log(5^2) = \log(50) - \log(25) = \log(\frac{50}{25}) = \log(2)$

**14** **a** $4 + {}^2\log(3) = {}^2\log(2^4) + {}^2\log(3) = {}^2\log(16) + {}^2\log(3) = {}^2\log(16 \cdot 3) = {}^2\log(48)$

**b** $3 + {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log((\frac{1}{2})^3) + {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log(\frac{1}{8}) + {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log(\frac{1}{8} \cdot 10) = {}^{\frac{1}{2}}\log(\frac{10}{8}) = {}^{\frac{1}{2}}\log(\frac{5}{4})$

**c** $5 - {}^3\log(5) = {}^3\log(3^5) - {}^3\log(5) = {}^3\log(243) - {}^3\log(5) = {}^3\log(\frac{243}{5})$

**d** $^2\log(12) - {}^3\log(9) = {}^2\log(12) - 2 = {}^2\log(12) - {}^2\log(2^2) = {}^2\log(12) - {}^2\log(4) = {}^2\log(\frac{12}{4}) = {}^2\log(3)$

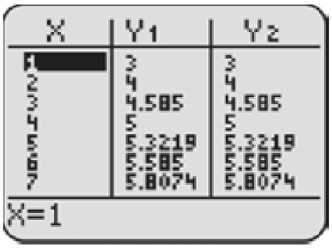
**e** $\frac{1}{2} \cdot {}^3\log(16) + {}^{\frac{1}{2}}\log(8) = {}^3\log(16^{\frac{1}{2}}) + {}^{\frac{1}{2}}\log((\frac{1}{2})^{-3}) = {}^3\log(4) - 3 = {}^3\log(4) - {}^3\log(3^3)$

$= {}^3\log(4) - {}^3\log(27) = {}^3\log(\frac{4}{27})$

**f** $\log(500) - {}^5\log(125) = \log(500) - {}^5\log(5^3) = \log(500) - 3 = \log(500) - \log(10^3) = \log(\frac{500}{1000}) = \log(\frac{1}{2})$

**15** **a** $\log(600) = \log(100 \cdot 6) = \log(100) + \log(6) = 2 + \log(6)$
**b** $^2\log(24) = {}^2\log(8 \cdot 3) = {}^2\log(8) + {}^2\log(3) = 3 + {}^2\log(3)$
**c** $^3\log(54) = {}^3\log(27 \cdot 2) = {}^3\log(27) + {}^3\log(2) = 3 + {}^3\log(2)$
**d** $^5\log(1250) = {}^5\log(625 \cdot 2) = {}^5\log(625) + {}^5\log(2) = 4 + {}^5\log(2)$

**16** **a**




$y_1 = y_2$

**b** $^2\log(8x) = {}^2\log(8) + {}^2\log(x) = 3 + {}^2\log(x)$

## bladzijde 14

**17** **a** $y = {}^2\log(x)$

$\downarrow$ verm. t.o.v. de $y$-as met $\frac{1}{32}$

$y = {}^2\log(32x)$

Er geldt $^2\log(32x) = {}^2\log(32) + {}^2\log(x) = 5 + {}^2\log(x)$.
Dus de translatie (0, 5) levert dezelfde beeldfiguur op.

**b** $y = {}^4\log(x)$

$\downarrow$ translatie $(0, \frac{1}{2})$

$y = {}^4\log(x) + \frac{1}{2}$

Er geldt $^4\log(x) + \frac{1}{2} = {}^4\log(x) + {}^4\log(4^{\frac{1}{2}}) = {}^4\log(x) + {}^4\log(2) = {}^4\log(2x)$

Dus de verm. t.o.v. de $y$-as met $\frac{1}{2}$ levert dezelfde beeldfiguur op.

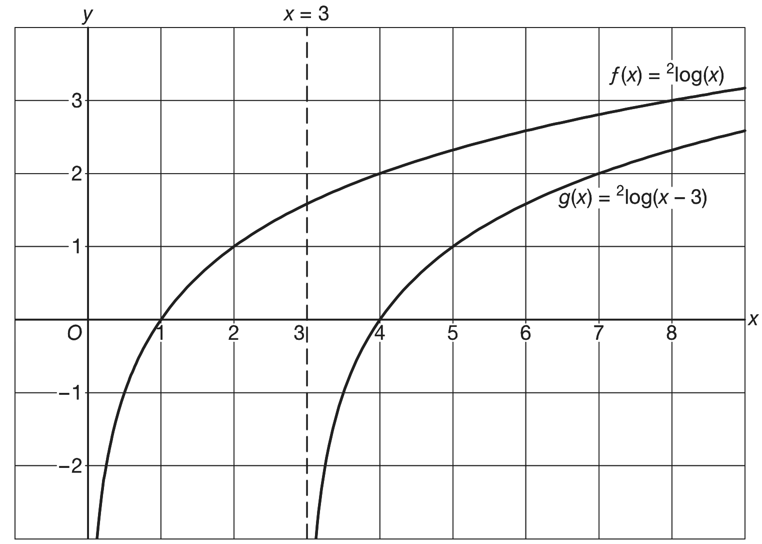
## bladzijde 15

**18** **a** $y = {}^2\log(x)$

$\downarrow$ translatie (3, 0)

$y = {}^2\log(x - 3)$
Dus $g(x) = {}^2\log(x - 3)$.

**b**




**c** Vermenigvuldiging t.o.v. de $y$-as is niet mogelijk omdat $(1, 0) \rightarrow (4, 0)$ en $(2, 1) \rightarrow (5, 1)$ niet met één vermenigvuldiging kan.
Een verticale translatie is niet mogelijk omdat de grafieken niet dezelfde asymptoot hebben.

**d** $y = {}^2\log(x - 3)$

$\downarrow$ verm. t.o.v. de $y$-as met $\frac{1}{4}$

$y = {}^2\log(4x - 3)$

Er geldt ${}^2\log(4x - 3) = {}^2\log(4(x - \frac{3}{4})) = {}^2\log(4) + {}^2\log(x - \frac{3}{4}) = 2 + {}^2\log(x - \frac{3}{4})$

$\left.\begin{array}{l} h(x) = 2 + {}^2\log(x - \frac{3}{4}) \\ h(x) = q + {}^2\log(x + p) \end{array}\right\} p = -\frac{3}{4} \wedge q = 2$

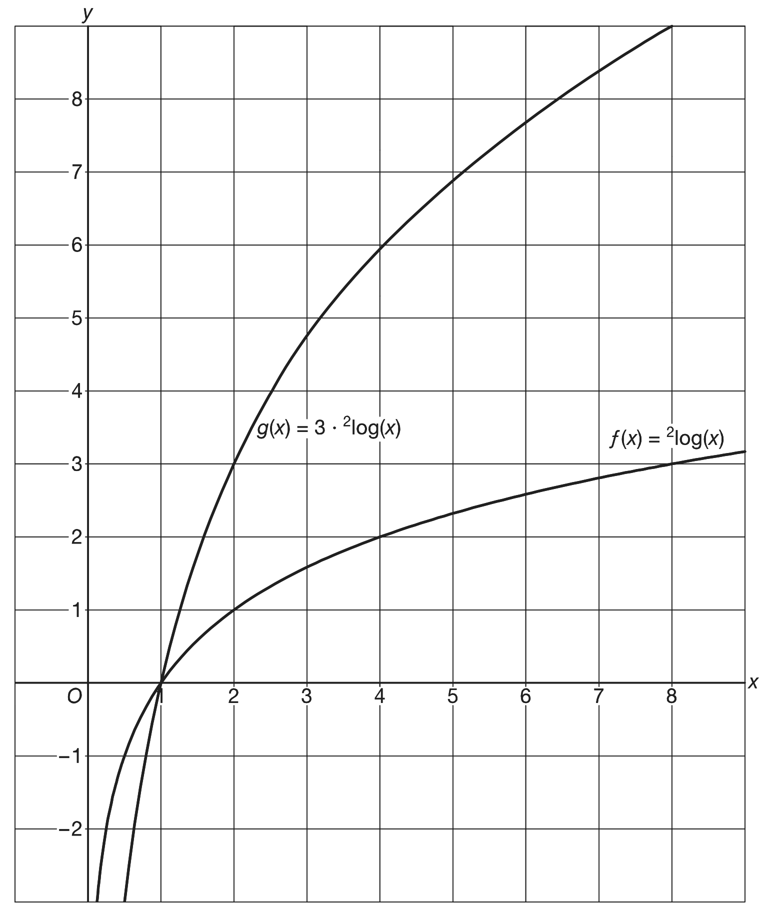
**19** **a** $y = {}^2\log(x)$

$\downarrow$ verm. t.o.v. de $x$-as met 3

$y = 3 \cdot {}^2\log(x)$

Dus $g(x) = 3 \cdot {}^2\log(x)$.

**b**



**c** Nee, de grafieken hebben niet dezelfde vorm.

**d** $y = 3 \cdot {}^2\log(x)$

$\downarrow$ translatie (1, 4)

$y = 3 \cdot {}^2\log(x - 1) + 4$

Er geldt $3 \cdot {}^2\log(x - 1) + 4 = {}^2\log((x - 1)^3) + {}^2\log(16) = {}^2\log(16(x - 1)^3)$

$\left.\begin{array}{l} h(x) = {}^2\log(16(x - 1)^3) \\ h(x) = {}^2\log(a(x - b)^c) \end{array}\right\} a = 16 \wedge b = 1 \wedge c = 3$

**20** **a** $y = {}^{\frac{1}{2}}\log(x)$

$\downarrow$ verm. t.o.v. de $y$-as met $\frac{1}{16}$

$y = {}^{\frac{1}{2}}\log(16x)$

De vermenigvuldiging t.o.v. de $y$-as met $\frac{1}{16}$.

**b** Er geldt ${}^{\frac{1}{2}}\log(16x) = {}^{\frac{1}{2}}\log(16) + {}^{\frac{1}{2}}\log(x) = -4 + {}^{\frac{1}{2}}\log(x)$.

Dus de translatie $(0, -4)$.

**c** Er geldt $5 + {}^{\frac{1}{2}}\log(x) = {}^{\frac{1}{2}}\log((\frac{1}{2})^5) + {}^{\frac{1}{2}}\log(x) = {}^{\frac{1}{2}}\log(\frac{1}{32}) + {}^{\frac{1}{2}}\log(x) = {}^{\frac{1}{2}}\log(\frac{1}{32}x)$.

Dus de vermenigvuldiging t.o.v. de $y$-as met 32.

**d** $g(x) = -4 + {}^{\frac{1}{2}}\log(x)$ (zie b)

$h(x) = 5 + {}^{\frac{1}{2}}\log(x)$

Dus de translatie (0, 9).

**e** $y = {}^{\frac{1}{2}}\log(16x)$

$\downarrow$ translatie (2, 3)

$y = {}^{\frac{1}{2}}\log(16(x - 2)) + 3$

Er geldt ${}^{\frac{1}{2}}\log(16(x-2)) + 3 = {}^{\frac{1}{2}}\log(16) + {}^{\frac{1}{2}}\log(x-2) + 3 = -4 + {}^{\frac{1}{2}}\log(x-2) + 3 = -1 + {}^{\frac{1}{2}}\log(x-2)$

$\left.\begin{array}{l} j(x) = -1 + {}^{\frac{1}{2}}\log(x-2) \\ j(x) = a + {}^{\frac{1}{2}}\log(x+b) \end{array}\right\} a = -1 \wedge b = -2$

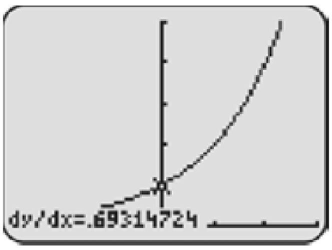
# 9.2 Het grondtal e

## bladzijde 17

**21** **a** $2^{x+h} - 2^x = 2^x \cdot 2^h - 2^x = 2^x(2^h - 1)$

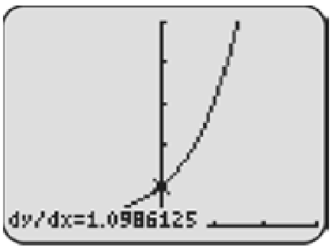
**b** $a^{x+h} - a^x = a^x \cdot a^h - a^x = a^x(a^h - 1)$

**22** **a**




**b** De optie d$y$/d$x$ (TI) of d/d$x$ (Casio) met $y_1 = 2^x$ geeft $\left[\frac{dy}{dx}\right]_{x=0} \approx 0{,}693$

**c**




De optie d$y$/d$x$ (TI) of d/d$x$ (Casio) met $y_1 = 3^x$ geeft $\left[\frac{dy}{dx}\right]_{x=0} \approx 1{,}099$.

**d** Uitproberen geeft $a \approx 2{,}72$.

## bladzijde 18

**23** **a**

| $h$ | 0,1 | 0,01 | 0,001 | 0,0001 | 0,00001 | 0,000001 |
|---|---|---|---|---|---|---|
| $a$ | 2,5937 | 2,7048 | 2,7169 | 2,7181 | 2,7183 | 2,7183 |

**b** Voor $a \approx 2{,}7183$.

## bladzijde 19

**24**
**a** $2e^2 - e^2 = e^2$
**b** $4\sqrt{e} - \sqrt{e} = 3\sqrt{e}$
**c** $5e^2 \cdot 3e^3 = 15e^5$
**d** $\frac{12e^6}{4e^2} = 3e^{6-2} = 3e^4$
**e** $e^{5x} \cdot e^x = e^{5x+x} = e^{6x}$
**f** $e^x \cdot e^2 = e^{x+2}$
**g** $5e^x - 3e^x = 2e^x$
**h** $(e^x + 1)^2 = (e^x)^2 + 2e^x + 1 = e^{2x} + 2e^x + 1$
**i** $\frac{6e^{2x} - e^x}{e^x} = \frac{6e^{2x}}{e^x} - \frac{e^x}{e^x} = 6e^x - 1$

**25**
**a** $(2x-4)e^x = 0$
$2x - 4 = 0 \vee e^x = 0$
$2x = 4$ geen opl.
$x = 2$
**b** $(x^2 - 3x) \cdot e^x = 0$
$x^2 - 3x = 0 \vee e^x = 0$
$x(x-3) = 0$ geen opl.
$x = 0 \vee x = 3$
**c** $x^2e^x = e^x$
$x^2e^x - e^x = 0$
$(x^2 - 1)e^x = 0$
$x^2 - 1 = 0 \vee e^x = 0$
$x^2 = 1$ geen opl.
$x = 1 \vee x = -1$
**d** $e^x + e^x = 2e^6$
$2e^x = 2e^6$
$e^x = e^6$
$x = 6$
**e** $e^x \cdot e^x = e^6$
$e^{2x} = e^6$
$2x = 6$
$x = 3$
**f** $\frac{e^{5x}}{e^x} = e$
$e^{4x} = e^1$
$4x = 1$
$x = \frac{1}{4}$

**26**
**a** $e^{3x} - e^x = 0$
$e^{3x} = e^x$
$3x = x$
$2x = 0$
$x = 0$
**b** $2x\,e^x + e^x = 0$
$(2x+1)e^x = 0$
$2x + 1 = 0 \vee e^x = 0$
$2x = -1$ geen opl.
$x = -\frac{1}{2}$
**c** $x^2 \cdot e^x - x \cdot e^x = 0$
$(x^2 - x)e^x = 0$
$x^2 - x = 0 \vee e^x = 0$
$x(x-1) = 0$ geen opl.
$x = 0 \vee x = 1$
**d** $e^{x+2} - \sqrt{e} = 0$
$e^{x+2} = \sqrt{e}$
$e^{x+2} = e^{\frac{1}{2}}$
$x + 2 = \frac{1}{2}$
$x = -1\frac{1}{2}$
**e** $e^{4x} - 1 = 0$
$e^{4x} = 1$
$e^{4x} = e^0$
$4x = 0$
$x = 0$
**f** $e^{2x-1} - e\sqrt{e} = 0$
$e^{2x-1} = e\sqrt{e}$
$e^{2x-1} = e^{1\frac{1}{2}}$
$2x - 1 = 1\frac{1}{2}$
$2x = 2\frac{1}{2}$
$x = 1\frac{1}{4}$

## bladzijde 20

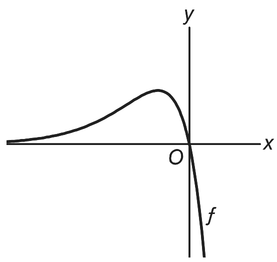
**27** $(1+h)^{\frac{1}{h}} \xrightarrow{h=\frac{1}{n}} (1+\frac{1}{n})^n$
$n$ heel groot geeft $\frac{1}{n}$ heel klein

$(1+h)^{\frac{1}{h}}$ met $h$ heel klein komt op hetzelfde neer als $(1+\frac{1}{n})^n$ met $n$ heel groot.

## bladzijde 21

**28**
**a** $f(x) = e^x + 2$ geeft $f'(x) = e^x$
**b** $f(x) = 2e^x + x^2$ geeft $f'(x) = 2e^x + 2x$
**c** $f(x) = x\,e^x + 4$ geeft $f'(x) = 1 \cdot e^x + x \cdot e^x = (x+1)e^x$
**d** $f(x) = (2x-4)e^x$ geeft $f'(x) = 2e^x + (2x-4)e^x = (2x-2)e^x$

**29** **a**




**b** $f(x) = -x\,e^x$ geeft $f'(x) = -1 \cdot e^x + (-x) \cdot e^x = (-x-1)e^x$
$f'(0) = -1 \cdot e^0 = -1$
De raaklijn is de lijn $y = -x$.

**c** $f'(x) = 0$ geeft $(-x-1)e^x = 0$
$-x - 1 = 0 \vee e^x = 0$
$-x = 1$ geen opl.
$x = -1$
$f(-1) = --1 \cdot e^{-1} = e^{-1} = \frac{1}{e}$
De top is $\left(1, \frac{1}{e}\right)$.

**30** **a** $f(x) = x^2\,e^x$ geeft $f'(x) = 2x \cdot e^x + x^2 \cdot e^x = (x^2 + 2x)e^x$
$f'(x) = 0$ geeft $(x^2 + 2x) \cdot e^x = 0$
$x^2 + 2x = 0 \vee e^x = 0$
$x(x + 2) = 0$ geen opl.
$x = 0 \vee x = -2$
max. is $f(-2) = 4e^{-2} = \frac{4}{e^2}$
min. is $f(0) = 0$

**b** $f(x) = g(x)$ geeft $x^2\,e^x = e^x$
$x^2\,e^x - e^x = 0$
$(x^2 - 1)e^x = 0$
$x^2 - 1 = 0 \vee e^x = 0$
$x^2 = 1$ geen opl.
$x = 1 \vee x = -1$
$f(1) = e$ en $f(-1) = e^{-1} = \frac{1}{e}$
$A(1, e)$ en $B\left(-1, \frac{1}{e}\right)$

**31** **a** $e^{2x} = e^{x+x} = e^x \cdot e^x$
**b** $f(x) = e^x \cdot e^x$ geeft $f'(x) = e^x \cdot e^x + e^x \cdot e^x = e^{2x} + e^{2x} = 2e^{2x}$

## bladzijde 23

**32** **a** Stel $y = e^{2x^2-3x} = e^u$ met $u = 2x^2 - 3x$.
$f'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot (4x-3) = (4x-3)e^{2x^2-3x}$

**b** Stel $y = e^{2x} = e^u$ met $u = 2x$.
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot 2 = 2e^{2x}$
$f'(x) = [4x^2]' \cdot e^{2x} + 4x^2 \cdot [e^{2x}]' = 8x \cdot e^{2x} + 4x^2 \cdot 2e^{2x} = (4x^2 + 8x)e^{2x}$

**c** Stel $y = 550e^{0{,}4x-0{,}6x^2} = 550e^u$ met $u = 0{,}4x - 0{,}6x^2$.
$f'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 550e^u \cdot (0{,}4 - 1{,}2x) = 550(0{,}4 - 1{,}2x)e^{0{,}4x-0{,}6x^2}$

**d** Stel $y = e^{3x} = e^u$ met $u = 3x$.
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot 3 = 3e^{3x}$
$f'(x) = [2x]' \cdot e^{3x} + 2x \cdot [e^{3x}]' = 2e^{3x} + 2x \cdot 3e^{3x} = 2e^{3x} + 6x \cdot e^{3x} = (6x+2)e^{3x}$

**33** Stel $y = e^{ax+b} = e^u$ met $u = ax + b$.
$[e^{ax+b}]' = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot a = a \cdot e^{ax+b}$

**34** **a** Stel $e^{-x^2} = e^u$ met $u = -x^2$.

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot -2x = -2x\, e^{-x^2}$

$f'(x) = [2x]' \cdot e^{-x^2} + 2x \cdot [e^{-x^2}]' = 2e^{-x^2} + 2x \cdot -2x\, e^{-x^2} = (-4x^2+2)e^{-x^2}$

**b** $f'(x) = 0$ geeft $(-4x^2+2)e^{-x^2} = 0$

$-4x^2 + 2 = 0 \;\vee\; e^{-x^2} = 0$

$-4x^2 = -2$ geen opl.

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}} \vee x = -\sqrt{\frac{1}{2}}$

max. is $f(\sqrt{\frac{1}{2}}) = 2\sqrt{\frac{1}{2}} \cdot e^{-\frac{1}{2}}$

**c** $2\sqrt{\frac{1}{2}} \cdot e^{-\frac{1}{2}} = \sqrt{4} \cdot \sqrt{\frac{1}{2}} \cdot \frac{1}{e^{\frac{1}{2}}} = \sqrt{2} \cdot \frac{1}{\sqrt{e}} = \frac{\sqrt{2}}{\sqrt{e}} = \sqrt{\frac{2}{e}}$

**d** min. is $f(-\sqrt{\frac{1}{2}}) = 2 \cdot -\sqrt{\frac{1}{2}} \cdot e^{-\frac{1}{2}} = -\sqrt{4} \cdot \sqrt{\frac{1}{2}} \cdot \frac{1}{e^{\frac{1}{2}}} = -\sqrt{2} \cdot \frac{1}{\sqrt{e}} = -\frac{\sqrt{2}}{\sqrt{e}} = -\sqrt{\frac{2}{e}}$

**35** **a** $f(x) = 8x\, e^{0,5x}$ geeft $f'(x) = 8 \cdot e^{0,5x} + 8x \cdot 0,5 \cdot e^{0,5x} = (4x+8)e^{0,5x}$

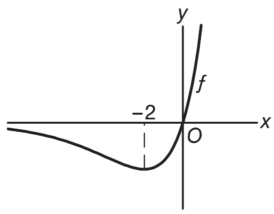
$f'(x) = 0$ geeft $(4x+8)e^{0,5x} = 0$

$4x + 8 = 0 \;\vee\; e^{0,5x} = 0$

$4x = -8$ geen opl.

$x = -2$

min. is $f(-2) = 8 \cdot -2 \cdot e^{-1} = -16e^{-1} = -\frac{16}{e}$




**b** Stel $k$: $y = ax + b$.

$a = f'(-4) = (-16+8) \cdot e^{-2} = -\frac{8}{e^2}$

$k$: $y = -\frac{8}{e^2}x + b$

$f(-4) = -32e^{-2} = -\frac{32}{e^2}$, dus $A\left(-4, -\frac{32}{e^2}\right)$ $\Big\}$ $-\frac{32}{e^2} = -\frac{8}{e^2} \cdot -4 + b$

$-\frac{32}{e^2} = \frac{32}{e^2} + b$

$-\frac{64}{e^2} = b$

Dus $k$: $y = -\frac{8}{e^2}x - \frac{64}{e^2}$.

**36** **a** Stel $y = e^{-\sqrt{x}} = e^u$ met $u = -\sqrt{x} = -x^{\frac{1}{2}}$.

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot -\frac{1}{2}x^{-\frac{1}{2}} = \frac{-1}{2x^{\frac{1}{2}}} \cdot e^{-\sqrt{x}} = -\frac{e^{-\sqrt{x}}}{2\sqrt{x}}$

$f'(x) = [x]' \cdot e^{-\sqrt{x}} + x \cdot [e^{-\sqrt{x}}]' = 1 \cdot e^{-\sqrt{x}} + x \cdot -\frac{e^{-\sqrt{x}}}{2\sqrt{x}} = e^{-\sqrt{x}} - \frac{x\, e^{-\sqrt{x}}}{2\sqrt{x}}$

Stel $l$: $y = ax + b$.

$a = f'(1) = e^{-1} - \frac{1 \cdot e^{-1}}{2 \cdot 1} = e^{-1} - \frac{1}{2}e^{-1} = \frac{1}{2}e^{-1} = \frac{1}{2e}$

$l$: $y = \frac{1}{2e}x + b$

$f(1) = 1 \cdot e^{-1} = \frac{1}{e}$ dus $A\left(1, \frac{1}{e}\right)$ $\Big\}$ $\frac{1}{e} = \frac{1}{2e} \cdot 1 + b$

$\frac{1}{e} - \frac{1}{2e} = b$

$\frac{2}{2e} - \frac{1}{2e} = b$

$\frac{1}{2e} = b$

Dus $l$: $y = \frac{1}{2e}x + \frac{1}{2e}$.

**b** $f'(x) = 0$ geeft $e^{-\sqrt{x}} - \dfrac{xe^{-\sqrt{x}}}{2\sqrt{x}} = 0$

$$\left(1 - \frac{x}{2\sqrt{x}}\right)e^{-\sqrt{x}} = 0$$

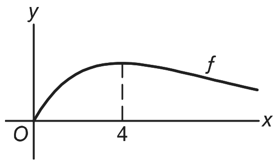
$$1 - \frac{x}{2\sqrt{x}} = 0 \ \vee \ e^{-\sqrt{x}} = 0$$

$1 - \frac{1}{2}\sqrt{x} = 0$ geen opl.

$\frac{1}{2}\sqrt{x} = 1$

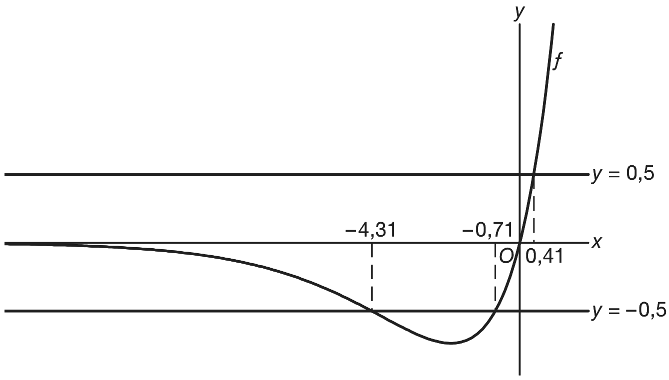
$\sqrt{x} = 2$

$x = 4$




max. is $f(4) = 4 \cdot e^{-2} = \dfrac{4}{e^2}$

**37** **a** Voer in $y_1 = x\,e^{0,5x}$, $y_2 = -0,5$ en $y_3 = 0,5$.
Intersect met $y_1$ en $y_2$ geeft $x \approx -4,31$ en $x \approx -0,71$
Intersect met $y_1$ en $y_3$ geeft $x \approx 0,41$




$-0,5 < f(x) < 0,5$ geeft $x < -4,31 \ \vee \ -0,71 < x < 0,41$

**b** $f(x) = g(x)$ geeft $x\,e^{0,5x} = x^2\,e^{0,5x}$

$x\,e^{0,5x} - x^2\,e^{0,5x} = 0$

$(x - x^2)e^{0,5x} = 0$

$x - x^2 = 0 \ \vee \ e^{0,5x} = 0$

$x(1 - x) = 0$ geen opl.

$x = 0 \ \vee \ x = 1$

$f(0) = 0$ en $f(1) = 1 \cdot e^{0,5} = \sqrt{e}$

De snijpunten zijn (0, 0) en $(1, \sqrt{e})$.

**c** $f(x) = x\,e^{0,5x}$ geeft $f'(x) = 1 \cdot e^{0,5x} + x \cdot 0,5 \cdot e^{0,5x} = (0,5x + 1)e^{0,5x}$
Stel $k$: $y = ax + b$.
$a = f'(4) = 3e^2$

$\left.\begin{array}{l} y = 3e^2x + b \\ f(4) = 4e^2, \text{ dus } A(4, 4e^2) \end{array}\right\}$ $4e^2 = 3e^2 \cdot 4 + b$

$4e^2 = 12e^2 + b$

$-8e^2 = b$

Dus $k$: $y = 3e^2x - 8e^2$.

**d** $g(x) = x^2\,e^{0,5x}$ geeft $g'(x) = 2x\,e^{0,5x} + x^2 \cdot 0,5 \cdot e^{0,5x} = (0,5x^2 + 2x)e^{0,5x}$
$g'(x) = 0$ geeft $(0,5x^2 + 2x)e^{0,5x} = 0$

$0,5x^2 + 2x = 0 \ \vee \ e^{0,5x} = 0$

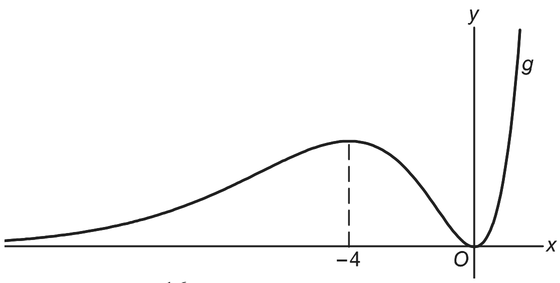
$x(0,5x + 2) = 0$ geen opl.

$x = 0 \ \vee \ 0,5x + 2 = 0$

$x = 0 \ \vee \ 0,5x = -2$

$x = 0 \ \vee \ x = -4$

$g(0) = 0$ en $g(-4) = 16e^{-2} = \dfrac{16}{e^2}$




max. is $g(-4) = \frac{16}{e^2}$

min. is $g(0) = 0$

**38** **a** Stel $y = e^{0,1x^3} = e^u$ met $u = 0,1x^3$.

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot 0,3x^2 = 0,3x^2\, e^{0,1x^3}$

$f(x) = x\, e^{0,1x^3}$ geeft $f'(x) = 1 \cdot e^{0,1x^3} + x \cdot 0,3x^2\, e^{0,1x^3} = (0,3x^3 + 1)e^{0,1x^3}$

Stel $k$: $y = ax + b$.

$a = f'(1) = (0,3 + 1)e^{0,1} = 1,3e^{0,1}$

$\left.\begin{array}{l} y = 1,3e^{0,1} + b \\ f(1) = 1 \cdot e^{0,1} = e^{0,1} \text{ dus } P(1, e^{0,1}) \end{array}\right\}$ $e^{0,1} = 1,3e^{0,1} \cdot 1 + b$

$-0,3e^{0,1} = b$

Dus $k$: $y = 1,3e^{0,1x} - 0,3e^{0,1}$.

**b** Stel $y = e^{ax^3} = e^u$ met $u = ax^3$.

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot 3ax^2 = 3ax^2\, e^{ax^3}$

$f(x) = x\, e^{ax^3}$ geeft $f'(x) = 1 \cdot e^{ax^3} + x \cdot 3ax^2\, e^{ax^3} = (3ax^3 + 1)e^{ax^3}$

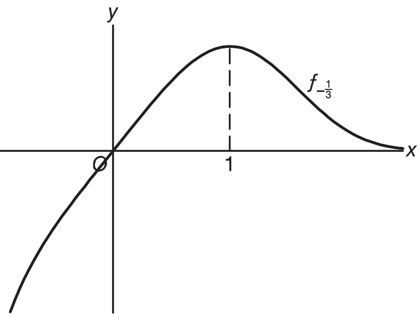
$f'(1) = 0$ geeft $(3a \cdot 1^3 + 1) \cdot e^{a \cdot 1^3} = 0$

$(3a + 1) \cdot e^a = 0$

$3a + 1 = 0 \vee e^a = 0$

$3a = -1$ geen opl.

$a = -\frac{1}{3}$




Voor $a = -\frac{1}{3}$ heeft $f_a$ een maximum voor $x = 1$.

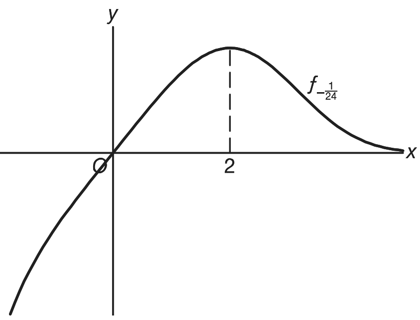
**c** $f'(2) = 0$ geeft $(3a \cdot 2^3 + 1) \cdot e^{a \cdot 2^3} = 0$

$(24a + 1) \cdot e^{8a} = 0$

$24a + 1 = 0 \vee e^{8a} = 0$

$24a = -1$ geen opl.

$a = -\frac{1}{24}$




Voor $a = -\frac{1}{24}$ heeft $f_a$ een maximum voor $x = 2$.

# 9.3 De natuurlijke logaritme

## bladzijde 25

**39** a $3^x = 20$
$x = {}^3\log(20)$
b $e^x = 20$
$x = {}^e\log(20)$

## bladzijde 26

**40** a $\ln(e) = 1$
b $\ln(e\sqrt{e}) = \ln(e \cdot e^{\frac{1}{2}}) = \ln(e^{1\frac{1}{2}}) = 1\frac{1}{2}$
c $\ln\left(\frac{1}{e}\right) = \ln(e^{-1}) = -1$
d $\ln(1) = \ln(e^0) = 0$
e $3\ln(e \cdot \sqrt[3]{e}) = 3\ln(e \cdot e^{\frac{1}{3}}) = 3\ln(e^{1\frac{1}{3}}) = 3 \cdot 1\frac{1}{3} = 4$
f $e^{\ln(7)} + e^{\ln(8)} = 7 + 8 = 15$
g $e^{2\ln(5)} = e^{\ln(5^2)} = e^{\ln(25)} = 25$
h $e^{\frac{1}{2}\ln(6)} = e^{\ln(6^{\frac{1}{2}})} = e^{\ln(\sqrt{6})} = \sqrt{6}$

**41** a $2\ln(3) + \ln(4) = \ln(3^2) + \ln(4) = \ln(9) + \ln(4) = \ln(9 \cdot 4) = \ln(36)$
b $\ln(20) - 3\ln(2) = \ln(20 - \ln(2^3) = \ln(20) - \ln(8) = \ln(\frac{20}{8}) = \ln(2\frac{1}{2})$
c $-2\ln(4) + \ln(12) = \ln(4^{-2}) + \ln(12) = \ln(\frac{1}{16}) + \ln(12) = \ln(\frac{12}{16}) = \ln(\frac{3}{4})$
d $4 + \ln(3) = \ln(e^4) + \ln(3) = \ln(3e^4)$
e $5 - \ln(5) = \ln(e^5) - \ln(5) = \ln\left(\frac{e^5}{5}\right)$
f $1 + \ln(10) = \ln(e) + \ln(10) = \ln(10e)$
g $\frac{1}{2} + 2\ln(6) = \ln(e^{\frac{1}{2}}) + \ln(6^2) = \ln(\sqrt{e}) + \ln(36) = \ln(36\sqrt{e})$
h $-3 + 4\ln(\frac{1}{2}) = \ln(e^{-3}) + \ln((\frac{1}{2})^4) = \ln\left(\frac{1}{e^3}\right) + \ln(\frac{1}{16}) = \ln\left(\frac{1}{16e^3}\right)$
i $e + \ln(2) = \ln(e^e) + \ln(2) = \ln(2e^e)$

**42** a $\ln(x) = 2$
$x = e^2$
b $\ln(x) = -1$
$x = e^{-1} = \frac{1}{e}$
c $2\ln(x) = 5$
$\ln(x) = 2\frac{1}{2}$
$x = e^{2\frac{1}{2}}$
d $\ln(3x) = 3$
$3x = e^3$
$x = \frac{1}{3}e^3$
e $\ln(x + 1) = 2$
$x + 1 = e^2$
$x = e^2 - 1$
f $\ln^2(x) = 16$
$\ln(x) = 4 \vee \ln(x) = -4$
$x = e^4 \vee x = e^{-4}$
g $(2x - 5)\ln(x) = 0$
$2x - 5 = 0 \vee \ln(x) = 0$
$2x = 5 \vee x = e^0$
$x = 2\frac{1}{2} \vee x = 1$
h $x\ln(x + 2) = 0$
$x = 0 \vee \ln(x + 2) = 0$
$x = 0 \vee x + 2 = e^0$
$x = 0 \vee x + 2 = 1$
$x = 0 \vee x = -1$
i $x\ln(x) = 0$
$x = 0 \vee \ln(x) = 0$
$x = 0 \vee x = e^0$
vold. niet $x = 1$

**43** a $e^x = 3$
$x = \ln(3)$
b $e^{3x} = 12$
$3x = \ln(12)$
$x = \frac{1}{3}\ln(12)$
c $5e^{2x} = 60$
$e^{2x} = 12$
$2x = \ln(12)$
$x = \frac{1}{2}\ln(12)$
d $4e^{1-x} = 20$
$e^{1-x} = 5$
$1 - x = \ln(5)$
$-x = -1 + \ln(5)$
$x = 1 - \ln(5)$

**e** $6 + e^{0,5x} = 10$
$e^{0,5x} = 4$
$0,5x = \ln(4)$
$x = 2\ln(4)$

**f** $\frac{3}{e^{2x}} = 10$
$10e^{2x} = 3$
$e^{2x} = \frac{3}{10}$
$2x = \ln(\frac{3}{10})$
$x = \frac{1}{2}\ln(\frac{3}{10})$

**44** **a** $\ln(x)(\ln(x) - 1) = 0$
$\ln(x) = 0 \vee \ln(x) - 1 = 0$
$x = 1 \vee \ln(x) = 1$
$x = 1 \vee x = e$

**b** $2\ln(x) = \ln(\frac{1}{2}) - \ln(2)$
$2\ln(x) = \ln(\frac{1}{4})$
$\ln(x) = \frac{1}{2}\ln(\frac{1}{4})$
$\ln(x) = \ln((\frac{1}{4})^{\frac{1}{2}})$
$\ln(x) = \ln(\sqrt{\frac{1}{4}})$
$\ln(x) = \ln(\frac{1}{2})$
$x = \frac{1}{2}$

**c** $e^{x^2} = 100$
$x^2 = \ln(100)$
$x = \sqrt{\ln(100)} \vee x = -\sqrt{\ln(100)}$

**d** $5e\,x + 2 = \ln\left(\frac{1}{e^3}\right)$
$5e\,x = -2 + \ln(e^{-3})$
$5e\,x = -2 - 3$
$5e\,x = -5$
$x = \frac{-5}{5e} = -\frac{1}{e}$

**e** $2\ln(x) + 3\ln(x) = 5 - \ln(32)$
$5\ln(x) = 5 - \ln(2^5)$
$5\ln(x) = 5 - 5\ln(2)$
$\ln(x) = 1 - \ln(2)$
$\ln(x) = \ln(e) - \ln(2)$
$\ln(x) = \ln\left(\frac{e}{2}\right)$
$x = \frac{1}{2}e$

**f** $\ln^2(x) - 2\ln(x) = 0$
$\ln(x) \cdot (\ln(x) - 2) = 0$
$\ln(x) = 0 \vee \ln(x) = 2$
$x = 1 \vee x = e^2$

## bladzijde 27

**45** **a** $2^x = e^{\ln(2^x)} = e^{x \cdot \ln(2)}$

**b** Stel $y = e^{x \cdot \ln(2)} = e^u$ met $u = x \cdot \ln(2)$.
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot \ln(2) = 2^x \cdot \ln(2)$

**46** **a** $f(x) = 4^x$ geeft $f'(x) = 4^x \cdot \ln(4)$

**b** $f(x) = 5^{x+2} = 5^u$ met $u = x + 2$ geeft
$f'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 5^u \cdot \ln(5) \cdot 1 = 5^{x+2} \cdot \ln(5)$

**c** $f(x) = 5 \cdot 6^x$ geeft $f'(x) = 5 \cdot 6^x \cdot \ln(6)$

**d** $f(x) = 3 \cdot 10^{2x} = 3 \cdot 10^u$ met $u = 2x$ geeft
$f'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 3 \cdot 10^u \cdot \ln(10) \cdot 2 = 6 \cdot 10^{2x} \cdot \ln(10)$

**e** $f(x) = (3x - 1) \cdot 2^x$ geeft $f'(x) = 3 \cdot 2^x + (3x - 1) \cdot 2^x \cdot \ln(2)$

**f** $f(x) = x^3 \cdot 3^x$ geeft $f'(x) = 3x^2 \cdot 3^x + x^3 \cdot 3^x \cdot \ln(3)$

**47** Bij opgave 22b was $f'(0) \approx 0,693$ en $\ln(2) \approx 0,693$.
Bij opgave 22c was $f'(0) \approx 1,099$ en $\ln(3) \approx 1,099$.

## bladzijde 28

**48** **a** Stel $y = 2^{2x} = 2^u$ met $u = 2x$.
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2^u \cdot \ln(2) \cdot 2 = 2^{2x} \cdot \ln(2) \cdot 2$
$f(x) = 2^{2x} - 2^x$ geeft $f'(x) = 2^{2x} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2)$
$f'(x) = 0$ geeft $2^{2x} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2) = 0$
$2^x \cdot \ln(2) \cdot (2 \cdot 2^x - 1) = 0$
$2^x \cdot \ln(2) = 0 \vee 2 \cdot 2^x - 1 = 0$
geen opl. $\quad 2^x = \frac{1}{2}$
$x = -1$
min. is $f(-1) = 2^{-2} - 2^{-1} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

**b** Stel $k$: $y = ax + b$.
$a = f'(1) = 2^2 \cdot \ln(2) \cdot 2 - 2^1 \cdot \ln(2) = 6\ln(2)$
$$\left.\begin{array}{l} y = 6\ln(2)\,x + b \\ f(1) = 2^2 - 2^1 = 4 - 2 = 2 \text{ dus } A(1, 2) \end{array}\right\} \begin{array}{l} 2 = 6\ln(2) \cdot 1 + b \\ 2 - 6\ln(2) = b \end{array}$$
Dus $k$: $y = 6\ln(2)\,x + 2 - 6\ln(2)$.

**49** **a** Stel $y = 2^{x^2} = 2^u$ met $u = x^2$.
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2^u \cdot \ln(2) \cdot 2x = 2^{x^2} \cdot \ln(2) \cdot 2x$$
$f(x) = 2^{x^2} - 2^x$ geeft $f'(x) = 2^{x^2} \cdot \ln(2) \cdot 2x - 2^x \cdot \ln(2)$
$f'(0{,}6) = 2^{0{,}36} \cdot \ln(2) \cdot 1{,}2 - 2^{0{,}6} \cdot \ln(2) \approx 0{,}017 \neq 0$
De grafiek van $f$ heeft geen top voor $x = 0{,}6$.

**b** Stel $k$: $y = ax$.
$a = f'(0) = 2^0 \cdot \ln(2) \cdot 2 \cdot 0 - 2^0 \cdot \ln(2) = -\ln(2)$
Dus $k$: $y = -\ln(2)\,x$.

Stel $l$: $y = ax + b$.
$a = f'(1) = 2 \cdot \ln(2) \cdot 2 \cdot 1 - 2 \cdot \ln(2) = 2\ln(2)$
$$\left.\begin{array}{l} y = 2\ln(2)\,x + b \\ A(1, 0) \end{array}\right\} \begin{array}{l} 0 = 2\ln(2) \cdot 1 + b \\ -2\ln(2) = b \end{array}$$
Dus $l$: $y = 2\ln(2)x - 2\ln(2)$.

**c** $2\ln(2)\,x - 2\ln(2) = -\ln(2)x$
$3\ln(2)\,x = 2\ln(2)$
$3x = 2$
$x = \frac{2}{3}$
Dus $x_B = \frac{2}{3}$.

**d** Voer in $y_1 = 2^{x^2} - 2x$.
De optie intersect met $y_1$ en $y_2 = 3$ geeft $x_C \approx -1{,}33$ en $x_D \approx 1{,}61$.
De optie intersect met $y_1$ en $y_2 = 4$ geeft $x_C \approx -1{,}46$ en $x_D \approx 1{,}69$.
Voor $p = 3$ is $CD \approx 1{,}61 - -1{,}33 = 2{,}94 < 3$ en voor $p = 4$ is $CD \approx 1{,}69 - -1{,}46 = 3{,}15 > 3$.
De kleinste waarde van $p$ is dus 4.

**50** **a**

| $x$ | 0,1 | 0,2 | 0,4 | 0,5 | 1 | 2 | 4 | 5 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| $y_2$ | 10 | 5 | 2,5 | 2 | 1 | 0,5 | 0,25 | 0,2 | 0,1 |

**b** Vermoeden: $f(x) = \ln(x)$ geeft $f'(x) = \frac{1}{x}$.

**c** Stel $y = e^{\ln(x)} = e^u$ met $u = \ln(x)$.
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot [\ln(x)]' = e^{\ln(x)} \cdot [\ln(x)]'$$
Dus $e^{\ln(x)} \cdot [\ln(x)]' = 1$
$x \cdot [\ln(x)]' = 1$
$[\ln(x)]' = \frac{1}{x}$

**51** **a** $g(x) = {}^2\log(x) = \frac{\ln(x)}{\ln(2)} = \frac{1}{\ln(2)} \cdot \ln(x)$ geeft $g'(x) = \frac{1}{\ln(2)} \cdot \frac{1}{x} = \frac{1}{x\ln(2)}$.

**b** $h(x) = {}^3\log(x) = \frac{\ln(x)}{\ln(3)} = \frac{1}{\ln(3)} \cdot \ln(x)$ geeft $h'(x) = \frac{1}{\ln(3)} \cdot \frac{1}{x} = \frac{1}{x\ln(3)}$.

## bladzijde 29

**52** I Stel $y = \ln(2x) = \ln(u)$ met $u = 2x$.
$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot 2 = \frac{1}{2x} \cdot 2 = \frac{1}{x}$$
II $f(x) = \ln(2x) = \ln(2) + \ln(x)$ geeft $f'(x) = \frac{1}{x}$
Manier II heeft de voorkeur.

**53** **a** $f(x) = \ln(3x) = \ln(3) + \ln(x)$ geeft $f'(x) = \frac{1}{x}$

**b** $f(x) = \ln(x^3) = 3\ln(x)$ geeft $f'(x) = 3 \cdot \frac{1}{x} = \frac{3}{x}$

**c** $f(x) = \ln(\sqrt{x}) = \ln(x^{\frac{1}{2}}) = \frac{1}{2}\ln(x)$ geeft $f'(x) = \frac{1}{2} \cdot \frac{1}{x} = \frac{1}{2x}$

**d** $f(x) = \ln\left(\dfrac{1}{x}\right) = \ln(x^{-1}) = -\ln(x)$ geeft $f'(x) = -\dfrac{1}{x}$

**e** $f(x) = \ln\left(\dfrac{1}{x^2}\right) = \ln(x^{-2}) = -2\ln(x)$ geeft $f'(x) = -2 \cdot \dfrac{1}{x} = -\dfrac{2}{x}$

**f** $y = \ln(2x - 5) = \ln(u)$ met $u = 2x - 5$ geeft

$$f'(x) = \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{u}\cdot 2 = \frac{2}{u} = \frac{2}{2x-5}$$

## bladzijde 30

**54** **a** $f(x) = {}^2\log(3x) = {}^2\log(3) + {}^2\log(x)$ geeft $f'(x) = \dfrac{1}{x\ln(2)}$

**b** $f(x) = {}^3\log(4x) = {}^3\log(4) + {}^3\log(x)$ geeft $f'(x) = \dfrac{1}{x\ln(3)}$

**c** $f(x) = {}^2\log\left(\dfrac{1}{x}\right) = {}^2\log(x^{-1}) = -{}^2\log(x)$ geeft $f'(x) = -\dfrac{1}{x\ln(2)}$

**d** $f(x) = {}^{\frac{1}{2}}\log(x^2) = 2\cdot {}^{\frac{1}{2}}\log(x)$ geeft $f'(x) = 2\cdot\dfrac{1}{x\ln(\frac{1}{2})} = \dfrac{2}{x\ln(\frac{1}{2})}$

**e** $f(x) = \log(5x - 6) = \log(u)$ met $u = 5x - 6$ geeft

$$f'(x) = \frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = \frac{1}{u\ln(10)}\cdot 5 = \frac{5}{u\ln(10)} = \frac{5}{(5x-6)\ln(10)}$$

**f** $f(x) = \log(2x) + \log(3x) = \log(2) + \log(x) + \log(3) + \log(x)$

geeft $f'(x) = \dfrac{1}{x\ln(10)} + \dfrac{1}{x\ln(10)} = \dfrac{2}{x\ln(10)}$

**55** **a** $f(x) = x^2 \cdot \ln(x)$ geeft $f'(x) = 2x\cdot\ln(x) + x^2\cdot\dfrac{1}{x} = 2x\ln(x) + x$

**b** $f(x) = e^x \cdot \ln(5x) = e^x\cdot(\ln(5) + \ln(x))$ geeft

$f'(x) = e^x\cdot(\ln(5) + \ln(x)) + e^x\cdot\dfrac{1}{x} = e^x\cdot\ln(5x) + \dfrac{e^x}{x}$

**c** $f(x) = x\log(x)$ geeft $f'(x) = 1\cdot\log(x) + x\cdot\dfrac{1}{x\ln(10)} = \log(x) + \dfrac{1}{\ln(10)}$

**56** **a** $f(x) = \ln(e^x) = x$ geeft $f'(x) = 1$

**b** $f(x) = {}^g\log(g^x) = x$ geeft $f'(x) = 1$

**c** $f(x) = e^{\ln(x)} = x\ (x > 0)$ geeft $f'(x) = 1\ (x > 0)$

**d** $f(x) = g^{{}^g\log(x)} = x\ (x > 0)$ geeft $f'(x) = 1\ (x > 0)$

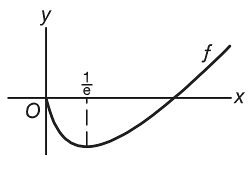
**57** **a** $f(x) = x\ln(x)$ geeft $f'(x) = 1\cdot\ln(x) + x\cdot\dfrac{1}{x} = \ln(x) + 1$

$f'(x) = 0$ geeft $\ln(x) + 1 = 0$

$\ln(x) = -1$

$x = e^{-1} = \dfrac{1}{e}$

min. is $f\left(\dfrac{1}{e}\right) = \dfrac{1}{e}\cdot\ln\left(\dfrac{1}{e}\right) = \dfrac{1}{e}\ln(e^{-1}) = \dfrac{1}{e}\cdot -1 = -\dfrac{1}{e}$




**b** Stel $k$: $y = ax + b$.

$a = f'(1) = \ln(1) + 1 = 1$

$y = x + b$

$f(1) = 1\cdot\ln(1) = 0$, dus $A(1, 0)$ $\Big\}$ $0 = 1 + b$

$-1 = b$

Dus $k$: $y = x - 1$.

**58** **a** $x - 2 > 0$ geeft $x > 2$, dus $D_f = \langle 2, \rightarrow\rangle$.

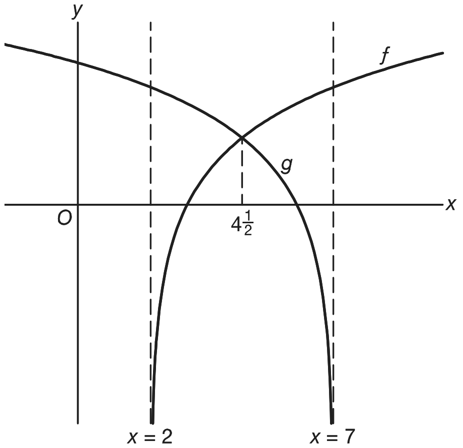
$7 - x > 0$ geeft $x < 7$, dus $D_g = \langle\leftarrow, 7\rangle$.

**b** $f(x) = g(x)$ geeft $\ln(x - 2) = \ln(7 - x)$

$x - 2 = 7 - x$

$2x = 9$

$x = 4\frac{1}{2}$




$f(x) \leq g(x)$ geeft $2 < x \leq 4\frac{1}{2}$

**c** $s(x) = \ln(x-2) + \ln(7-x)$ geeft

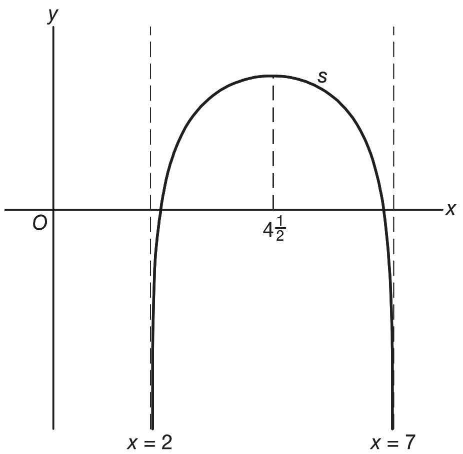
$s'(x) = \frac{1}{x-2} \cdot 1 + \frac{1}{7-x} \cdot -1 = \frac{1}{x-2} - \frac{1}{7-x}$

$s'(x) = 0$ geeft $\frac{1}{x-2} = \frac{1}{7-x}$

$x - 2 = 7 - x$

$2x = 9$

$x = 4\frac{1}{2}$




max. is $s(4\frac{1}{2}) = \ln(2\frac{1}{2}) + \ln(2\frac{1}{2}) = 2\ln(2\frac{1}{2})$

**59** **a** $3 - x > 0 \wedge x + 2 > 0$

$-x > -3 \wedge x > -2$

$x < 3 \wedge x > -2$

$D_f = \langle -2, 3 \rangle$.

**b** Stel $y = \ln(3-x) = \ln(u)$ met $u = 3 - x$.

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot -1 = -\frac{1}{3-x}$

Stel $y = \ln(x+2) = \ln(u)$ met $u = x + 2$.

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot 1 = \frac{1}{x+2}$

$f(x) = 2\ln(3-x) + \ln(x+2)$ geeft $f'(x) = 2 \cdot -\frac{1}{3-x} + \frac{1}{x+2} = \frac{-2}{3-x} + \frac{1}{x+2}$

$f'(x) = 0$ geeft $\frac{-2}{3-x} + \frac{1}{x+2} = 0$

$\frac{1}{x+2} = \frac{2}{3-x}$

$2x + 4 = 3 - x$

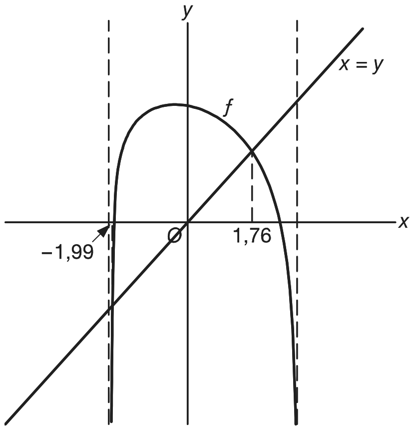
$3x = -1$

$x = -\frac{1}{3}$

$f(-\frac{1}{3}) = 2 \cdot \ln(3\frac{1}{3}) + \ln(1\frac{2}{3}) = 2\ln(\frac{10}{3}) + \ln(\frac{5}{3}) = \ln((\frac{10}{3})^2) + \ln(\frac{5}{3}) = \ln(\frac{100}{9} \cdot \frac{5}{3}) = \ln(\frac{500}{27})$

De top van de grafiek is $(-\frac{1}{3}, \ln(\frac{500}{27}))$.

**c** Voer in $y_1 = 2\ \ln(3 - x) + \ln(x + 2)$ en $y_2 = x$.
De optie intersect geeft $x \approx -1{,}99$ en $x \approx 1{,}76$.




$f(x) > x$ geeft $-1{,}99 < x < 1{,}76$

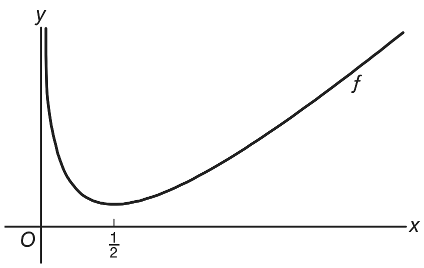
**60** **a** $f(x) = 2x - \ln(4x) = 2x - \ln(4) - \ln(x)$ geeft $f'(x) = 2 - \dfrac{1}{x}$

$f'(x) = 0$ geeft $2 - \dfrac{1}{x} = 0$

$-\dfrac{1}{x} = -2$

$x = \frac{1}{2}$

$f(\frac{1}{2}) = 2 \cdot \frac{1}{2} - \ln(4 \cdot \frac{1}{2}) = 1 - \ln(2) = \ln(\mathrm{e}) - \ln(2) = \ln\left(\dfrac{\mathrm{e}}{2}\right) = \ln(\frac{1}{2}\mathrm{e})$




min. is $f(\frac{1}{2}) = \ln(\frac{1}{2}\mathrm{e})$

**b** $f'(x) = \frac{1}{2}$ geeft $2 - \dfrac{1}{x} = \frac{1}{2}$

$-\dfrac{1}{x} = -1\frac{1}{2}$

$\dfrac{1}{x} = \dfrac{3}{2}$

$x = \frac{2}{3}$

$f(\frac{2}{3}) = 2 \cdot \frac{2}{3} - \ln(4 \cdot \frac{2}{3}) = \frac{4}{3} - \ln(\frac{8}{3})$

Dus $A(\frac{2}{3}, \frac{4}{3} - \ln(\frac{8}{3}))$.

**c** $f'(x) = 2$ geeft $2 - \dfrac{1}{x} = 2$

$-\dfrac{1}{x} = 0$

geen opl.

De grafiek heeft geen raaklijn met rc = 2, want $f'(x) = 2$ heeft geen oplossing.

# 9.4 Grafieken en formules

**bladzijde 32**

**61** **a** $g(x) = \mathrm{e}^{x+3}$

**b** Er geldt $\mathrm{e}^{x+3} = \mathrm{e}^x \cdot \mathrm{e}^3 = \mathrm{e}^3 \cdot \mathrm{e}^x$.
De grafiek van $g$ ontstaat uit die van $f$ bij de vermenigvuldiging t.o.v. de $x$-as met $\mathrm{e}^3$.

**62** **a** $g(x) = \ln(x) + 3$.
**b** Er geldt $\ln(x) + 3 = \ln(x) + \ln(e^3) = \ln(e^3 \cdot x)$.
De grafiek van $g$ ontstaat uit die van $f$ bij de vermenigvuldiging t.o.v. de $y$-as met $\frac{1}{e^3}$.

## bladzijde 33

**63** **a** $y = e^x$
$\downarrow$ translatie $(-1, 0)$
$y = e^{x+1}$
$y = e^{x+1} = e^x \cdot e^1 = e \cdot e^x$
Dus de vermenigvuldiging t.o.v. de $x$-as met e levert dezelfde beeldgrafiek op.
**b** $y = e^x$
$\downarrow$ verm. t.o.v. de $x$-as met $\frac{1}{2}$
$y = \frac{1}{2} \cdot e^x$
$y = \frac{1}{2} \cdot e^x = e^{\ln(\frac{1}{2})} \cdot e^x = e^{\ln(\frac{1}{2})+x} = e^{x+\ln(\frac{1}{2})}$
Dus de translatie $(-\ln(\frac{1}{2}), 0)$ levert dezelfde beeldgrafiek op.

**64** **a** $y = \ln(x)$
$\downarrow$ translatie $(0, 4)$
$y = \ln(x) + 4$
$y = \ln(x) + 4 = \ln(x) + \ln(e^4) = \ln(x \cdot e^4) = \ln(e^4 \cdot x)$
Dus de vermenigvuldiging t.o.v. de $y$-as met $\frac{1}{e^4}$ levert dezelfde beeldgrafiek op.
**b** $y = \ln(x)$
$\downarrow$ verm. t.o.v. de $y$-as met $\frac{1}{4}$
$y = \ln(4x)$
$y = \ln(4x) = \ln(4) + \ln(x) = \ln(x) + \ln(4)$
Dus de translatie $(0, \ln(4))$ levert dezelfde beeldgrafiek op.

**65** $y = 4\,e^x$
$\downarrow$ translatie $(\ln(2), 0)$
$y = 4\,e^{x-\ln(2)}$
$\downarrow$ verm. t.o.v. de $x$-as met 6
$y = 6 \cdot 4\,e^{x-\ln(2)} = 24\,e^{x-\ln(2)}$
$y = 24\,e^{x-\ln(2)} = \frac{24\,e^x}{e^{\ln(2)}} = \frac{24\,e^x}{2} = 12\,e^x$
$\left.\begin{array}{l} y = 12\,e^x \\ y = a \cdot e^x \end{array}\right\} a = 12$

**66** $y = 2\ln(x)$
$\downarrow$ translatie $(0, \ln(3))$
$y = 2\ln(x) + \ln(3)$
$\downarrow$ verm. t.o.v. de $y$-as met $\frac{1}{4}$
$y = 2\ln(4x) + \ln(3)$
$y = 2\ln(4x) + \ln(3) = \ln((4x)^2) + \ln(3) = \ln(16x^2) + \ln(3) = \ln(48x^2)$
$\left.\begin{array}{l} y = \ln(48x^2) \\ y = \ln(ax^b) \end{array}\right\} a = 48 \wedge b = 2$

**67** **a** $y = \ln(x + 1)$
$\ln(x + 1) = y$
$x + 1 = e^y$
$x = -1 + e^y$
**b** $y = \frac{1}{2}e^x$
$\frac{1}{2}e^x = y$
$e^x = 2y$
$x = \ln(2y)$

**68** **a** $y = 10\,e^{2x}$
$10\,e^{2x} = y$
$e^{2x} = \frac{1}{10}y$
$2x = \ln(\frac{1}{10}y)$
$x = \frac{1}{2}\ln(\frac{1}{10}y)$

**b** $y = \ln(2x - 5)$
$\ln(2x - 5) = y$
$2x - 5 = e^y$
$2x = 5 + e^y$
$x = 2\frac{1}{2} + \frac{1}{2}e^y$

**c** $y = 4 \cdot 3^{5x}$
$4 \cdot 3^{5x} = y$
$3^{5x} = \frac{1}{4}y$
$5x = {}^3\log(\frac{1}{4}y)$
$x = \frac{1}{5} \cdot {}^3\log(\frac{1}{4}y)$

**d** $y = {}^2\log(4x - 2)$
${}^2\log(4x - 2) = y$
$4x - 2 = 2^y$
$4x = 2 + 2^y$
$x = \frac{1}{2} + \frac{1}{4} \cdot 2^y$

**e** $y = 0{,}1e^{4x-1}$
$0{,}1e^{4x-1} = y$
$e^{4x-1} = 10y$
$4x - 1 = \ln(10y)$
$4x = 1 + \ln(10y)$
$x = \frac{1}{4} + \frac{1}{4}\ln(10y)$

**f** $y = 5\ln(1 - 10x)$
$5\ln(1 - 10x) = y$
$\ln(1 - 10x) = \frac{1}{5}y$
$1 - 10x = e^{\frac{1}{5}y}$
$-10x = -1 + e^{\frac{1}{5}y}$
$x = \frac{1}{10} - \frac{1}{10}e^{\frac{1}{5}y}$

**69** a $y = 50 \cdot 2^x$
$\ln(y) = \ln(50 \cdot 2^x) = \ln(50) + \ln(2^x) = \ln(50) + x \cdot \ln(2)$
$\left.\begin{array}{l}\ln(y) = \ln(50) + x \cdot \ln(2)\\ \ln(y) = \ln(a) + x \cdot \ln(b)\end{array}\right\}\ a = 50 \ \wedge \ b = 2$

**b** $y = 50 \cdot 2^x$
$\log(y) = \log(50 \cdot 2^x) = \log(50) + \log(2^x) = \log(50) + x \cdot \log(2) = x \cdot \log(2) + \log(50)$
$\left.\begin{array}{l}\log(y) = x \cdot \log(2) + \log(50)\\ \log(y) = x \cdot \log(p) + \log(q)\end{array}\right\}\ p = 2 \ \wedge \ q = 50$

**c** $y = 50 \cdot 2^x$
${}^2\log(y) = {}^2\log(50 \cdot 2^x)$
${}^2\log(y) = {}^2\log(50) + {}^2\log(2^x)$
${}^2\log(y) = {}^2\log(50) + x$
${}^2\log(y) = x + {}^2\log(50)$
Dus $y = 50 \cdot 2^x$ is te herleiden tot ${}^2\log(y) = x + {}^2\log(50)$.

**70** **a** $y = 100 \cdot 9^x$
$\ln(y) = \ln(100 \cdot 9^x)$
$\ln(y) = \ln(100) + \ln(9^x)$
$\ln(y) = \ln(100) + x \cdot \ln(9)$
$\ln(y) = x \cdot \ln(9) + \ln(100)$

**b** $y = 100 \cdot 9^x$
$\log(y) = \log(100 \cdot 9^x)$
$\log(y) = \log(100) + \log(9^x)$
$\log(y) = \log(100) + x \cdot \log(9)$
$\log(y) = x \cdot \log(9) + \log(100)$

**c** $y = 100 \cdot 9^x$
${}^3\log(y) = {}^3\log(100 \cdot 9^x)$
${}^3\log(y) = {}^3\log(100) + {}^3\log(9^x)$
${}^3\log(y) = {}^3\log(100) + x \cdot {}^3\log(9)$
${}^3\log(y) = {}^3\log(100) + x \cdot 2$
${}^3\log(y) = 2x + {}^3\log(100)$

**71** **a** $y = \ln(x) - 2$
$\ln(x) - 2 = y$
$\ln(x) = 2 + y$
$x = e^{2+y}$
$x = e^2 \cdot e^y$

**b** $y = {}^2\log(x) - 3$
${}^2\log(x) - 3 = y$
${}^2\log(x) = 3 + y$
$x = 2^{3+y}$
$x = 2^3 \cdot 2^y$
$\left.\begin{array}{l}x = 8 \cdot 2^y\\ x = b \cdot 2^y\end{array}\right\}\ b = 8$

**c** $y = 0{,}5\ \log(x) - 2$
$0{,}5\ \log(x) - 2 = y$
$0{,}5\ \log(x) = 2 + y$
$\log(x) = 4 + 2y$
$x = 10^{4+2y}$
$x = 10^4 \cdot 10^{2y}$
$x = 10\,000 \cdot (10^2)^y$
$\left.\begin{array}{l} x = 10\,000 \cdot 100^y \\ x = b \cdot g^y \end{array}\right\}\ b = 10\,000 \ \wedge\ g = 100$

**72** **a** $y = 2\ \ln(x) - 5$
$2\ \ln(x) - 5 = y$
$2\ \ln(x) = 5 + y$
$\ln(x) = 2\frac{1}{2} + \frac{1}{2}y$
$x = \mathrm{e}^{2\frac{1}{2}+\frac{1}{2}y}$
$x = \mathrm{e}^{2\frac{1}{2}} \cdot \mathrm{e}^{\frac{1}{2}y}$
$x = \mathrm{e}^2 \cdot \sqrt{\mathrm{e}} \cdot (\mathrm{e}^{\frac{1}{2}})^y$
$x = \mathrm{e}^2 \cdot \sqrt{\mathrm{e}} \cdot (\sqrt{\mathrm{e}})^y$
Dus $b = \mathrm{e}^2 \cdot \sqrt{\mathrm{e}}$ en $g = \sqrt{\mathrm{e}}$.

**b** $y = 10 \cdot {}^3\log(x) - 4$
$10 \cdot {}^3\log(x) - 4 = y$
$10 \cdot {}^3\log(x) = 4 + y$
${}^3\log(x) = 0{,}4 + 0{,}1y$
$x = 3^{0{,}4+0{,}1y}$
$x = 3^{0{,}4} \cdot 3^{0{,}1y}$
$x = 3^{\frac{2}{5}} \cdot (3^{\frac{1}{10}})^y$
$x = \sqrt[5]{3^2} \cdot (\sqrt[10]{3})^y$
Dus $b = \sqrt[5]{9}$ en $g = \sqrt[10]{3}$.

**c** $y = 5\ \log(2x) - 6$
$5\ \log(2x) - 6 = y$
$5\ \log(2x) = 6 + y$
$\log(2x) = 1\frac{1}{5} + \frac{1}{5}y$
$2x = 10^{1\frac{1}{5}+\frac{1}{5}y}$
$x = \frac{1}{2} \cdot 10^{1\frac{1}{5}+\frac{1}{5}y}$
$x = \frac{1}{2} \cdot 10^{1\frac{1}{5}} \cdot 10^{\frac{1}{5}y}$
$x = \frac{1}{2} \cdot 10^{1\frac{1}{5}} \cdot (10^{\frac{1}{5}})^y$
$x = \frac{1}{2} \cdot 10\sqrt[5]{10} \cdot (\sqrt[5]{10})^y = 5\sqrt[5]{10} \cdot (\sqrt[5]{10})^y$
Dus $b = 5\sqrt[5]{10}$ en $g = \sqrt[5]{10}$.

## bladzijde 35

**73** $N = 100 \cdot 1{,}2^t$
$\ln(N) = \ln(100 \cdot 1{,}2^t)$
$\ln(N) = \ln(100) + \ln(1{,}2^t)$
$\ln(N) = \ln(100) + t \cdot \ln(1{,}2)$
$\ln(N) = \ln(1{,}2) \cdot t + \ln(100)$
$a = \ln(1{,}2) \approx 0{,}182$ en $b = \ln(100) \approx 4{,}605$

**74** **a** $P = 5{,}6\ \ln(N) - 25$
$5{,}6\ \log(N) - 25 = P$
$5{,}6\ \log(N) = P + 25$
$\log(N) = \dfrac{P}{5{,}6} + \dfrac{25}{5{,}6}$
$N = 10^{\frac{P}{5{,}6}+\frac{25}{5{,}6}} = 10^{\frac{P}{5{,}6}} \cdot 10^{\frac{25}{5{,}6}} = 10^{\frac{25}{5{,}6}} \cdot (10^{\frac{1}{5{,}6}})^P$
Dus $N \approx 29\,000 \cdot 1{,}51^P$.

**b** Je gebruikt $\ln(N) = \dfrac{\log(N)}{\log(\mathrm{e})}$.

$P = 5{,}6\ \ln(N) - 25 = 5{,}6 \cdot \dfrac{\log(N)}{\log(\mathrm{e})} - 25 = \dfrac{5{,}6}{\log(\mathrm{e})} \cdot \ln(N) - 25 \approx 12{,}9\ \ln(N) - 25$

**c** $P = 12{,}9 \log(N) - 25$

$12{,}9 \log(N) - 25 = P$

$12{,}9 \log(N) = 25 + P$

$\log(N) = \frac{25}{12{,}9} + \frac{P}{12{,}9}$

$N = 10^{\frac{25}{12{,}9} + \frac{P}{12{,}9}}$

$N = 10^{\frac{25}{12{,}9}} \cdot 10^{\frac{P}{12{,}9}}$

$N \approx 87 \cdot (10^{\frac{1}{12{,}9}})^P \approx 87 \cdot 1{,}20^P$

## bladzijde 36

**75** **a** $t = 12 \ln(M) - 16$

$12 \ln(M) - 16 = t$

$12 \ln(M) = 16 + t$

$\ln(M) = \frac{16}{12} + \frac{t}{12}$

$M = e^{\frac{16}{12} + \frac{t}{12}}$

$M = e^{\frac{16}{12}} \cdot (e^{\frac{1}{12}})^t$

$M \approx 3{,}79 \cdot 1{,}09^t$

**b** $S = 15 \log(R) - 20$

$15 \log(R) - 20 = S$

$15 \log(R) = 20 + S$

$\log(R) = \frac{20}{15} + \frac{S}{15}$

$R = 10^{\frac{20}{15} + \frac{S}{15}}$

$R = 10^{\frac{20}{15}} \cdot (10^{\frac{1}{15}})^S \approx 22 \cdot 1{,}17^S$

**76** **a** $y = 4\ e^{0{,}5x}$

$\downarrow$ translatie (ln(4), 0)

$y = 4\ e^{0{,}5(x - \ln(4))}$

$\downarrow$ verm. t.o.v. de $x$-as met 3

$y = 12\ e^{0{,}5(x - \ln(4))}$

$y = 12\ e^{0{,}5(x-\ln(4))} = 12\ e^{0{,}5x - 0{,}5\ln(4)} = 12\ e^{0{,}5x} \cdot e^{-0{,}5\ln(4)} = 12\ e^{0{,}5x} \cdot e^{\ln(4^{-\frac{1}{2}})} = 12\ e^{0{,}5x} \cdot e^{\ln(\frac{1}{\sqrt{4}})} = 12\ e^{0{,}5x} \cdot e^{\ln(\frac{1}{2})} = 12\ e^{0{,}5x} \cdot \frac{1}{2} = 6\ e^{0{,}5x}$

Dus het beeld van de grafiek van $f$ is $y = 6e^{0{,}5x}$.

**b** $y = 4\ e^{0{,}5x} = 4 \cdot 10^{\log(e^{0{,}5x})} = 4 \cdot 10^{0{,}5x \log(e)} \approx 4 \cdot 10^{0{,}22x}$

Dus $p = 4$ en $q \approx 0{,}22$.

**77** **a** $h = 130$ geeft $\log(W) = 0{,}008 \cdot 130 + 0{,}38$

$\log(W) = 1{,}42$

$W = 10^{1{,}42} \approx 26$

Dus zijn gewicht is 26 kg.

**b** $W = 23{,}5$ geeft $\log(23{,}5) = 0{,}008h + 0{,}38$

$0{,}008h + 0{,}38 = \log(23{,}5)$

$0{,}008h = \log(23{,}5) - 0{,}38$

$h = \frac{\log(23{,}5) - 0{,}38}{0{,}008} \approx 124$

Dus haar lengte is 1,24 m.

**c** $\log(W) = 0{,}008h + 0{,}38$

$W = 10^{0{,}008h + 0{,}38}$

$W = (10^{0{,}008})^h \cdot 10^{0{,}38}$

$W \approx 1{,}0186^h \cdot 2{,}40 = 2{,}40 \cdot 1{,}0186^h$

Dus $W = 2{,}40 \cdot 1{,}0186^h$.

**d** $W = 2{,}40 \cdot 1{,}0186^h = 2{,}40 \cdot e^{\ln(1{,}0186^h)} = 2{,}40 \cdot e^{h \cdot \ln(1{,}0186)} \approx 2{,}40 \cdot e^{h \cdot 0{,}0184} = 2{,}40 \cdot e^{0{,}0184h}$

Dus $W = 2{,}40 \cdot e^{0{,}0184h}$.

**78** **a** $D = 50$ geeft $\ln(N) = 12{,}1 - 1{,}7 \ln(50)$

$N = e^{12{,}1 - 1{,}7 \ln(50)} \approx 233$

Er staan 233 bomen per ha.

**b** $N = \dfrac{2000}{8} = 250$

$N = 250$ geeft $\ln(250) = 12{,}1 - 1{,}7\ln(D)$

$1{,}7\ln(D) = 12{,}1 - \ln(250)$

$\ln(D) = \dfrac{12{,}1 - \ln(250)}{1{,}7} \approx 3{,}87$

$D \approx e^{3{,}87} \approx 48$

De gemiddelde diameter is 48 cm.

**c** $\ln(N) = 12{,}1 - 1{,}7\ln(D)$

$1{,}7\ln(D) = 12{,}1 - \ln(N)$

$\ln(D) = \frac{12{,}1}{1{,}7} - \frac{1}{1{,}7}\ln(\quad)$

$D = e^{\frac{12{,}1}{1{,}7} - \frac{1}{1{,}7}\ln(\quad)}$

$D = e^{\frac{12{,}1}{1{,}7}} \cdot e^{-\frac{1}{1{,}7}\ln(\quad)}$

$D = e^{\frac{12{,}1}{1{,}7}} \cdot (e^{\ln(\quad)})^{-\frac{1}{1{,}7}}$

$D \approx 1230 \cdot \quad^{-\frac{1}{1{,}7}} \approx 1230 \cdot N^{-0{,}59}$

Dus $a = 1230$ en $b = -0{,}59$.

**79** **a** $L = 80$ geeft $T = -2{,}57\ln\left(\dfrac{87-80}{63}\right) \approx 5{,}65$ jaar $\approx$ 5 jaar en 8 maanden.

**b** $T = 10$ geeft $10 = -2{,}57\ln\left(\dfrac{87-\quad}{63}\right)$

$2{,}57\ln\left(\dfrac{87-\quad}{63}\right) = -10$

$\ln\left(\dfrac{87-\quad}{63}\right) = -\frac{10}{2{,}57}$

$\dfrac{87-\quad}{63} = e^{-\frac{10}{2{,}57}}$

$87 - L = 63 \cdot e^{-\frac{10}{2{,}57}}$

$L = 87 - 63\,e^{-\frac{10}{2{,}57}} \approx 85{,}7$ feet

De lengte is $85{,}7 \cdot 0{,}314 \approx 26{,}9$ m.

**c** $T = -2{,}57\ln\left(\dfrac{87-\quad}{63}\right)$

$2{,}57\ln\left(\dfrac{87-\quad}{63}\right) = -T$

$\ln\left(\dfrac{87-\quad}{63}\right) = \dfrac{-\quad}{2{,}57}$

$\dfrac{87-\quad}{63} = e^{-\frac{\quad}{2{,}57}}$

$87 - L = 63 \cdot e^{-\frac{\quad}{2{,}57}}$

$L = 87 - 63 \cdot e^{-\frac{1}{2{,}57}}$

$L = 87 - 63 \cdot e^{-\frac{1}{2{,}57}}$

$L \approx 87 - 63 \cdot e^{-0{,}39}$
$L = a + b\,e^{cT}$ $\Big\}$ $a = 87 \;\wedge\; b = -63 \;\wedge\; c \approx -0{,}39$

# Diagnostische toets

## bladzijde 38

**1** **a** $y = 3^x$

$\downarrow$ translatie (–2, 0)

$y = 3^{x+2}$

$y = 3^{x+2} = 3^x \cdot 3^2 = 3^x \cdot 9 = 9 \cdot 3^x$.

Dus de vermenigvuldiging t.o.v. de $x$-as met 9 levert dezelfde beeldgrafiek op.

**b** $y = 3^x$

$\downarrow$ verm. t.o.v. de $x$-as met $\frac{1}{3}$

$y = \frac{1}{3} \cdot 3^x$

$y = \frac{1}{3} \cdot 3^x = 3^{-1} \cdot 3^x = 3^{x-1}$

Dus de translatie (1, 0) levert dezelfde beeldgrafiek op.

**2** **a** ${}^2\log(9) + {}^2\log(11) = {}^2\log(9 \cdot 11) = {}^2\log(99)$

**b** $3 - {}^5\log(20) = {}^5\log(5^3) - {}^5\log(20) = {}^5\log(125) - {}^5\log(20) = {}^5\log(\frac{125}{20}) = {}^5\log(6\frac{1}{4})$

**c** $-2 + \log(18) = \log(10^{-2}) + \log(18) = \log(10^{-2} \cdot 18) = \log(\frac{18}{100}) = \log(\frac{9}{50})$

**d** ${}^{\frac{1}{2}}\log(8) + {}^3\log(54) = -3 + {}^3\log(54) = {}^3\log(3^{-3}) + {}^3\log(54) = {}^3\log(3^{-3} \cdot 54) = {}^3\log(2)$

**3** **a** $y = {}^3\log(x)$

$\downarrow$ translatie (0, –2)

$y = {}^3\log(x) - 2$

$y = {}^3\log(x) - 2 = {}^3\log(x) - {}^3\log(3^2) = {}^3\log(x) - {}^3\log(9) = {}^3\log\left(\frac{x}{9}\right)$

Dus de vermenigvuldiging t.o.v. de $y$-as met 9 levert dezelfde beeldgrafiek op.

**b** $y = {}^3\log(x)$

$\downarrow$ verm. t.o.v. de $y$-as met $\frac{1}{27}$

$y = {}^3\log(27x)$

$y = {}^3\log(27x) = {}^3\log(27) + {}^3\log(x) = {}^3\log(3^3) + {}^3\log(x) = 3 + {}^3\log(x) = {}^3\log(x) + 3$

Dus de translatie (0, 3) levert dezelfde beeldgrafiek op.

**4** $y = {}^2\log(4x)$

$\downarrow$ translatie (–2, 3)

$y = {}^2\log(4(x+2)) + 3$

$y = {}^2\log(4(x+2)) + 3 = {}^2\log(4) + {}^2\log(x+2) + 3 = 2 + {}^2\log(x+2) + 3 = 5 + {}^2\log(x+2)$

$\left.\begin{array}{l} g(x) = 5 + {}^2\log(x+2) \\ g(x) = a + {}^2\log(x+b) \end{array}\right\} a = 5 \wedge b = 2$

**5** **a** $3e^3 - 8e^3 = -5e^3$

**b** $5e^{5a} - 3e^{5a} = 2e^{5a}$

**c** $\dfrac{6\,e^{8x} + 3\,e^{4x}}{e^{2x}} = \dfrac{6\,e^{8x}}{e^{2x}} + \dfrac{3\,e^{4x}}{e^{2x}} = 6\,e^{6x} + 3\,e^{2x}$

**d** $(2\,e^a - 1)^2 = 4\,e^{2a} - 4\,e^a + 1$

**6** **a** $(x^2 - 4x - 5)e^x = 0$

$x^2 - 4x - 5 = 0 \vee e^x = 0$

$(x-5)(x+1) = 0$ geen opl.

$x = 5 \vee x = -1$

**b** $e^{2x+1} - 1 = 0$

$e^{2x+1} = 1$

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

**c** $e^{x+1} - e^2 \cdot \sqrt{e} = 0$

$e^{x+1} = e^2 \cdot \sqrt{e}$

$e^{x+1} = e^2 \cdot e^{\frac{1}{2}}$

$e^{x+1} = e^{2\frac{1}{2}}$

$x + 1 = 2\frac{1}{2}$

$x = 1\frac{1}{2}$

**d** $x^2\,e^{2x} = x\,e^{2x}$

$x^2\,e^{2x} - x\,e^{2x} = 0$

$(x^2 - x)e^{2x} = 0$

$x^2 - x = 0 \vee e^{2x} = 0$

$x(x-1) = 0$ geen opl.

$x = 0 \vee x = 1$

**7** **a** $f(x) = e^x + 2x^2 - 5x$ geeft $f'(x) = e^x + 4x - 5$

**b** $g(x) = e^{2\ {}^2 -5} = e^u$ met $u = 2x^2 - 5x$ geeft

$$g'(x) = \frac{d}{d} = \frac{d}{d} \cdot \frac{d}{d} = e^u \cdot (4x - 5) = e^{2\ {}^2 -5} \cdot (4x - 5)$$

**c** $h(x) = (2x^2 - 5x) \cdot e^x$ geeft
$h'(x) = (4x - 5) \cdot e^x + (2x^2 - 5x) \cdot e^x = (4x - 5 + 2x^2 - 5x) \cdot e^x = (2x^2 - x - 5)e^x$

**d** $j(x) = 2x^2\, e^x - 5x$ geeft $j'(x) = 4x\, e^x + 2x^2\, e^x - 5 = (2x^2 + 4x)e^x - 5$

**8** **a** Stel $y = e^{-x} = e^u$ met $u = -x$.

$$\frac{d}{d} = \frac{d}{d} \cdot \frac{d}{d} = e^u \cdot -1 = -e^{-x}$$

$f(x) = (2x - 3) \cdot e^{-x}$ geeft $f'(x) = 2 \cdot e^{-x} + (2x - 3) \cdot -e^{-x} = (2 - 2x + 3) \cdot e^{-x} = (-2x + 5)e^{-x}$

$f(x) = 0$ geeft $(2x - 3)e^x = 0$

$2x - 3 = 0 \lor e^x = 0$

$2x = 3$ geen opl.

$x = 1\frac{1}{2}$

Stel $k$: $y = ax + b$.

$$a = f'(1\tfrac{1}{2}) = (-2 \cdot 1\tfrac{1}{2} + 5) \cdot e^{-1\frac{1}{2}} = 2e^{-1\frac{1}{2}} = \frac{2}{e\sqrt{e}}$$

$$\left.\begin{array}{l} y = \frac{2}{e\sqrt{e}}x + b \\ \text{door } (1\frac{1}{2}, 0) \end{array}\right\} 0 = \frac{2}{e\sqrt{e}} \cdot 1\tfrac{1}{2} + b$$

$$-\frac{3}{e\sqrt{e}} = b$$

Dus $k$: $y = \frac{2}{e\sqrt{e}}x - \frac{3}{e\sqrt{e}}$.

**b** $f'(x) = 0$ geeft $(-2x + 5)e^{-x} = 0$

$-2x + 5 = 0 \lor e^{-x} = 0$

$-2x = -5$ geen opl.

$x = 2\frac{1}{2}$

$$f(2\tfrac{1}{2}) = (2 \cdot 2\tfrac{1}{2} - 3) \cdot e^{-2\frac{1}{2}} = 2e^{-2\frac{1}{2}} = \frac{2}{e^{2\frac{1}{2}}} = \frac{2}{e^2\sqrt{e}}$$

De top is het punt $\left(2\frac{1}{2}, \frac{2}{e^2\sqrt{e}}\right)$.

## bladzijde 39

**9** **a** $e^x = 4$

$x = \ln(4)$

**b** $e^{x-2} = 3$

$x - 2 = \ln(3)$

$x = 2 + \ln(3)$

**c** $e^{\frac{1}{2}} = 5$

$\frac{1}{2} = \ln(5)$

$x = 2\ln(5)$

**d** $\ln(x) = -2$

$x = e^{-2}$

$x = \frac{1}{e^2}$

**e** $\ln(2 - x) = 4$

$2 - x = e^4$

$-x = -2 + e^4$

$x = 2 - e^4$

**f** $\ln^2(x) = 25$

$\ln(x) = 5 \lor \ln(x) = -5$

$x = e^5 \lor x = e^{-5}$

$x = e^5 \lor x = \frac{1}{e^5}$

**g** $\ln(x^2) = 6$

$x^2 = e^6$

$x = e^3 \lor x = -e^3$

**h** $e^{2x} \cdot \ln(\tfrac{1}{2}x) = 0$

$e^{2x} = 0 \;\vee\; \ln(\tfrac{1}{2}x) = 0$

geen opl. $\tfrac{1}{2}x = 1$

$x = 2$

**i** $(x - e) \cdot \ln(x) = 0$

$x - e = 0 \;\vee\; \ln(x) = 0$

$x = e \;\vee\; x = 1$

**10** **a** $f(x) = 3^{4x-1} = 3^u$ met $u = 4x - 1$ geeft

$f'(x) = \dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = 3^u \cdot \ln(3) \cdot 4 = 3^{4x-1} \cdot \ln(3) \cdot 4$

**b** $g(x) = 5 \cdot 2^{3x} = 5 \cdot 2^u$ met $u = 3x$ geeft $g'(x) = 5 \cdot 2^{3x} \cdot \ln(2) \cdot 3 = 15 \cdot 2^{3x} \cdot \ln(2)$

**c** $h(x) = x^2 \cdot 4^x$ geeft $h'(x) = 2x \cdot 4^x + x^2 \cdot 4^x \cdot \ln(4)$

**11** **a** $f(x) = \ln(ex) = \ln(e) + \ln(x)$ geeft $f'(x) = \dfrac{1}{x}$

**b** $g(x) = e \cdot \ln(3x) = e(\ln(3) + \ln(x))$ geeft $g'(x) = e \cdot \dfrac{1}{x} = \dfrac{e}{x}$

**c** $h(x) = x^2 + \ln(x^2) = x^2 + 2\ln(x)$ geeft $h'(x) = 2x + 2 \cdot \dfrac{1}{x} = 2x + \dfrac{2}{x}$

**d** $j(x) = x^2 \cdot {}^2\log(x)$ geeft $j'(x) = 2x \cdot {}^2\log(x) + x^2 \cdot \dfrac{1}{x\ln(2)} = 2x \cdot {}^2\log(x) + \dfrac{x}{\ln(2)}$

**e** $k(x) = \ln\left(\dfrac{6}{x}\right) = \ln(6) - \ln(x)$ geeft $k'(x) = -\dfrac{1}{x}$

**f** $l(x) = {}^4\log(4x) = {}^4\log(4) + {}^4\log(x)$ geeft $l'(x) = \dfrac{1}{x\ln(4)}$

**12** **a** $D_f = \langle 0, \rightarrow\rangle$

**b** $f(x) = x^2 \cdot \ln(x)$ geeft $f'(x) = 2x \cdot \ln(x) + x^2 \cdot \dfrac{1}{x} = 2x\ln(x) + x$

$f(x) = 0$ geeft $x^2 \cdot \ln(x) = 0$

$x^2 = 0 \;\vee\; \ln(x) = 0$

geen opl. $x = 1$

Het snijpunt met de $x$-as is $(1, 0)$.

Stel $k$: $y = ax + b$.

$a = f'(1) = 2 \cdot 1 \cdot \ln(1) + 1 = 1$

$\left.\begin{array}{l} y = x + b \\ \text{door } (1, 0) \end{array}\right\}\; 0 = 1 + b$

$-1 = b$

Dus $k$: $y = x - 1$.

**c** $f'(x) = 0$ geeft $2x\ln(x) + x = 0$

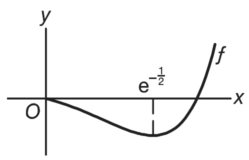
$x(2\ln(x) + 1) = 0$

$x = 0 \;\vee\; 2\ln(x) + 1 = 0$

vold. niet $\;2\ln(x) = -1$

$\ln(x) = -\tfrac{1}{2}$

$x = e^{-\frac{1}{2}}$




min. is $f(e^{-\frac{1}{2}}) = (e^{-\frac{1}{2}})^2 \cdot \ln(e^{-\frac{1}{2}}) = e^{-1} \cdot -\tfrac{1}{2} = -\dfrac{1}{2e}$

**13** **a** $y = e^x$

$\downarrow$ verm. t.o.v. de $x$-as met 2

$y = 2 \cdot e^x$

$y = 2\,e^x = e^{\ln(2)} \cdot e^x = e^{x + \ln(2)}$

Dus de translatie $(-\ln(2), 0)$ levert dezelfde beeldgrafiek op.

**b** $y = \ln(x)$

$\downarrow$ verm. t.o.v. de $y$-as met 2

$y = \ln(\tfrac{1}{2}x)$

$y = \ln(\tfrac{1}{2}x) = \ln(\tfrac{1}{2}) + \ln(x) = \ln(x) + \ln(\tfrac{1}{2})$

Dus de translatie $(0, \ln(\tfrac{1}{2}))$ levert dezelfde beeldgrafiek op.

**14** $y = 3\ln(x)$
$\downarrow$ translatie (0, 2)
$y = 3\ln(x) + 2$
$\downarrow$ verm. t.o.v. de $y$-as met 2
$y = 3\ln(\frac{1}{2}x) + 2$

$y = 3\ln(\frac{1}{2}x) + 2 = \ln((\frac{1}{2}x)^3) + \ln(e^2) = \ln(\frac{1}{8}x^3) + \ln(e^2) = \ln(\frac{1}{8}e^2x^3)$

$$\left.\begin{array}{l} g(x) = \ln(\frac{1}{8}e^2x^3) \\ g(x) = \ln(ax^b) \end{array}\right\} a = \frac{1}{8}e^2 \wedge b = 3$$

**15** $p = 18\ln(q) - 15$
$18\ln(q) - 15 = p$
$18\ln(q) = 15 + p$
$\ln(q) = \frac{15}{18} + \frac{p}{18}$
$q = e^{\frac{15}{18} + \frac{p}{18}}$
$q = e^{\frac{15}{18}} \cdot e^{\frac{p}{18}}$
$q = e^{\frac{15}{18}} \cdot (e^{\frac{1}{18}})^p \approx 2{,}301 \cdot 1{,}057^p$

**16** **a** $G = 100$ geeft $F = 16(0{,}6 + \ln(100)) \approx 83$
De hartslagfrequentie is 83 slagen per minuut.
**b** $F = 78$ geeft $78 = 16(0{,}6 + \ln(G))$
$16(0{,}6 + \ln(G)) = 78$
$0{,}6 + \ln(G) = \frac{78}{16}$
$\ln(G) = \frac{78}{16} - 0{,}6$
$G = e^{\frac{78}{16} - 0{,}6} \approx 72$
Een gewicht van 72 kg.
**c** $F = 16(0{,}6 + \ln(G))$
$16(0{,}6 + \ln(G)) = F$
$0{,}6 + \ln(G) = \frac{F}{16}$
$\ln(G) = \frac{F}{16} - 0{,}6$
$G = e^{\frac{F}{16} - 0{,}6}$
$G = e^{\frac{F}{16}} \cdot e^{-0{,}6}$
$G = e^{-0{,}6} \cdot (e^{\frac{1}{16}})^F \approx 0{,}549 \cdot 1{,}064^F$
Dus $G = 0{,}549 \cdot 1{,}064^F$.